VOL. V, FASC. 1 E 2, P.71-81

ON THE CORRECTION OF REVERSING THERMOMETERS AND CONSTRUCTION OF GRAPH FOR TOTAL CORRECTION

(RECEIVED MAY/4/1954)

Ingvar Emilsson

LIST OF SYMBOLS

T_w = water temperature in situ

T = reading of reversing thermometer

I = correction for index error on the scale of reversing thermometer

$T' = T + I$

t = temperature at which the reversing thermometer is read

$\Delta T = T_w - T'$

v_o = volume of mercury, expressed in °C, below the mark 0 °C on the scale of reversing thermometer

$\frac{1}{K}$ = coefficient of thermal expansion of the thermometer system. Usually $\frac{1}{6100}$

Other symbols are explained in the text.

I - GENERAL CONSIDERATIONS

1. DERIVATION OF FORMULAE - It is well known that readings of reversing thermometers have to be corrected as the torn-off column of mercury changes its volume when brought into a temperature which differs from that at reversal.

The change dv of the torn-off column is proportional to its volume v . We have therefore:

$$dv = \frac{1}{K}\, vdt \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Assuming K to be constant in the temperature range T_{v} to t we have:

$$\int_{T_{v}}^{t} \frac{dv}{v} = \frac{1}{K} \int_{T_{v}}^{t} dt$$

and therefore $$\log\left[\frac{T' + v_{\circ}}{T_{v} + v_{\circ}}\right] = \frac{t - T_{v}}{K} \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

or $$\log\left[\frac{T_{v} + v_{\circ}}{T' + v_{\circ}}\right] = \frac{T_{v} - t}{K} \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

where $T_{v} + v_{\circ}$ is the volume of the torn-off mercury column at reversal expressed in °C, and $T' + v_{\circ}$ the volume at reading.

Following Sverdrup (1947) we introduce $T' = T_{v} - \Delta T$ in (2) it then reads:

$$\log\left[1 - \frac{\Delta T}{T_{v} + v_{\circ}}\right] = -\frac{T_{v} - t}{K} \quad . \quad . \quad . \quad . \qquad (4)$$

By expanding the logarithm into series and taking only the terms of first order into consideration we get:

$$\Delta T = \frac{(T_{v} - t)(T_{v} + v_{\circ})}{K} \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

This expression has served as a basis for a number of simplified formulae for temperature corrections. As pointed out by Sverdrup (1947) the expression is not quite exact since terms of higher orders than the first are neglected.

Let us now consider (3). Eliminating T_{v} this equation reads:

$$\log\left[1 + \frac{\Delta T}{T' + v_{\circ}}\right] = \frac{T' - t}{K} + \frac{\Delta T}{K} \quad . \quad . \quad . \qquad (6)$$

which we shall write for the sake of convenience

$$\log(1 + X) = \frac{x'}{K} + X\frac{y'}{K} \quad . \quad . \quad . \quad . \qquad (7)$$

the symbols having the following meaning:

$$y' = T' + v_o \, ; \; x' = T - t$$
$$X = \frac{\Delta T}{T' + v} = \frac{\Delta T}{y'}$$

The transcendental equation (7) can be solved by graphical method by drawing the two curves:

$$Y_1 = \log(1 + X)$$
$$Y_2 = \frac{x'}{K} + X\frac{y'}{K}$$

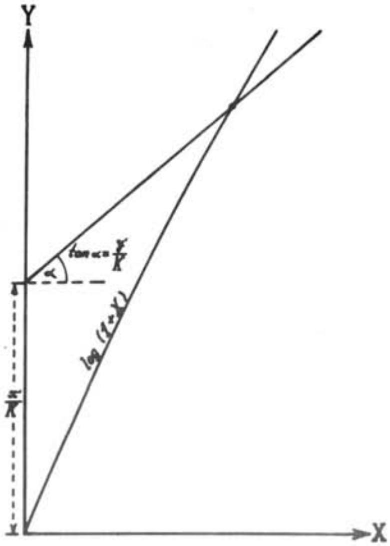

Fig. 1

In cartesian coordinates the first represents a logarithmic curve and the second a straight line intersecting the ordinate at $\frac{x'}{K}$ and having the angle coefficient $\frac{y'}{K}$ (see Fig. 1). The point of intersection will give a certain value of X corresponding to x' and y', and ΔT can be computed from the relation $\Delta T = Xy'$

Such a procedure, which has a degree of accuracy depending only upon the scale and exactness of the graph is, however, fairly tedious.

Expansion of the logarithm into series gives:

$$\log(1+X) = X - \frac{X^2}{2} + \frac{X^3}{3} - \frac{X^4}{4} + \cdots$$

Since X is always a small quantity $\left(0 \leq X \leq \frac{1}{50}\right)$ the series converge very quickly.

Let us now consider the expression $\frac{2X}{2+X}$. Expanding into power series:

$$\frac{2X}{2+X} = X - \frac{X^2}{2} + \frac{X^3}{4} - \frac{X^4}{8} + \cdots$$

and subtracting from the logarithmic series the difference of the first four terms is:

$$\frac{2X^3 - 3X^4}{24}$$

As this difference will never exceed 0.00000064 it is readily seen that the logarithmic term in (7) can be replaced by $\frac{2X}{2+X}$ without any loss of accuracy of the correction.

Since $X = \frac{\Delta T}{y'}$, this substitution gives:

$$\frac{\Delta T}{y' + \frac{\Delta T}{2}} = \frac{x'}{K} + \frac{\Delta T}{K} \quad . \quad . \quad . \quad . \quad (8)$$

which can further be written:

$$\Delta T = \frac{x'y'}{K - (y' + \frac{x'}{2})} + \frac{\Delta T^2}{2\left[K - (y' + \frac{x'}{2})\right]} \quad . \quad . \quad (9)$$

Neglecting the last term in (9) we get a formula which is identical with that given by W.Hansen (1934) and nearly identical with the formula given by Sverdrup (1947).*

Taking x' as a dependent and ΔT as an independent variable the equation (8) reads:

$$x' = \frac{K\Delta T}{y' + \frac{\Delta T}{2}} - \Delta T \quad . \quad . \quad . \quad . \quad (10)$$

(*) - Sverdrup writes x' instead of $\frac{x'}{2}$.

By means of this equation values of x' for given values of ΔT and y' can be computed and compiled into tables.

Recently LaFond (1951) published extensive tables for x' using the formula of Sverdrup (1947). Tables of this kind facilitate the construction of convenient graphs for correction of reversing thermometers.

In the case of unprotected thermometers T_u is already known. Equation (3) reads in this case:

$$\log\left[\frac{T_{w,u} + v_0}{T'_u + v_0}\right] = \frac{T_u - t_u}{K} \quad . \quad . \quad . \quad (11)$$

where symbols with the subscript u refer to unprotected thermometers.

We shall also write:

$$T_{w,u} = T'_u + \Delta T_u$$

Now eliminating $T_{w,u}$ (11) becomes:

$$\log\left[1 + \frac{\Delta T_u}{T'_u + v_0}\right] = \frac{T_u + t_u}{K} \quad . \quad . \quad . \quad (12)$$

By a procedure similar to those employed for protected thermometers we get:

$$\Delta T_u = \frac{x_u \, y'_u}{K - \frac{x_u}{2}} \quad . \quad . \quad . \quad . \quad (13)$$

where $x_u = T_u - t_u$ and $y'_u = T'_u + v_0$

When x_u is taken as dependent and T_u as an independent variable we can write:

$$x_u = \frac{K \, \Delta T_u}{y'_u + \frac{\Delta T_u}{2}} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This equation is well fitted for computing tables similar to those used for protected thermometers.

2. THE INDEX CORRECTION - The scale of most thermometers not being quite true, an index correction I has to be added to the thermometer reading.

From equation (10) we have:

$$x_1 = \frac{KI}{y + \frac{I}{2}} - I \quad . \quad . \quad . \quad . \quad (15)$$

where $y = T + v_0$ and x_1 is the value corresponding to the index correction I.

By the same equation we also have:

$$x_1 + x' = x_1 + x + I = \frac{KC}{y + \frac{C}{2}} - C \quad . \quad . \quad (16)$$

where $x = T - t$

Eliminating x_1 by (15) and (16):

$$x + \frac{KI}{y + \frac{I}{2}} = \frac{KC}{y + \frac{C}{2}} - C \quad . \quad . \quad . \quad (17)$$

To show that no error of importance is made by replacing C by $I + \Delta T$ a somewhat lengthy mathematical discussion is required, we shall therefore confine ourselves to numerical examples.

Let us first consider the exceptional case

$$x = 25° \; ; \; y = 200° \; ; \; I = 1.0°$$

Inserting into formulae (11) and (17) we get

$$\Delta T + I - C = 0.00020° \ldots..$$

Now considering the "normal" case

$$x = 10° \; ; \; y = 100° \; ; \; I = 0.1°$$

we analogously get

$$\Delta T + I - C = 0.00005° \ldots..$$

An index correction as high as 1.0 °C is never found for thermometers of good quality. It might occur for thermometers which have been broken and repaired. The index correction for most thermometers does not exceed 0.1 °C.

It is seen therefore that no error of importance is made by putting $C = \Delta T + I$ in equation (17).

For unprotected thermometers equation (17) takes the form:

$$x_u + \frac{KI}{y_u + \frac{I}{2}} = \frac{KC_u}{y + \frac{C_u}{2}} \qquad . \quad . \quad . \quad (18)$$

It can be shown that the error made by replacing C_u by $I + \Delta T_u$ is even less than for protected thermometers.

We shall in the following give a description of a graph for total correction of the reversing thermometers where the results above are applied.

II - CONSTRUCTION OF A CORRECTION GRAPH

1. CONSTRUCTION OF A MASTER-GRAPH FOR ALL REVERSING THERMOMETERS -

A paper of good quality has to be used. The ratio breadth/length of the sheet is suitably chosen as 1:5. A recommended scale is 1 °C corresponding to 1cm on the graph. The scale chosen depends upon the desired accuracy of the correction. With the above recommended scale, readings can easily be obtained with an error less than ± 0.005 °C.

A center line is drawn parallel to the longer edge of the sheet. This line will be called the ordinate. Along the ordinate values of y' are marked according to the chosen scale. For each integral value of y' lines are drawn at right angles to the ordinate (i.e. parallel to the abscissa). The computed values of x' are now measured along the abscissa corresponding to a given value of y' and the proper value of ΔT is noted at this point. The negative values of ΔT on the *right* and positive on the *left* hand side of the ordinate. Smooth curves are then drawn with China ink through the points $\Delta T =$ const.

Fig. 2 shows a portion of such master-graph constructed by means of the tables for protected thermometers given by LaFond (1951). Curves are drawn with an interval of 0.01 °C for ΔT starting at ± 0.005. Every second interval is shaded to facilitate reading.

2. PREPARATION OF CORRECTION GRAPH FOR AN INDIVIDUAL THERMOMETER - A direct photocopy is taken of a portion of the master-graph corresponding to the proper $v_{\circ}$ extending from $v_{\circ} + T'_{max}$ to $v_{\circ} + T'_{min}$, T'_{max}, T'_{min} being the extremes of the thermometer scale.

Values of T' are then written with China ink along the ordinate as shown in Fig. 3.

In order to add the index error I computed values of the expression $\frac{KI}{y + \frac{I}{2}}$ (see p. 76) are plotted along the abscissas corresponding to the value of y (not y') the positive to the *left* and negative to the *right* hand side of the ordinate. The points thus obtained are connected with a smooth *curve of index correction.*

Master and individual graphs for unprotected thermometers are constructed in a similar way using proper formulae or tables.

The correction graph has a marked scale only along the ordinate. Care should therefore be observed when making the photocopy to avoid deformation along the abscissa. When soaked the photographic paper expands mainly in one direction. The ordinate should therefore be given that direction. If allowed to dry slowly and freely in horizontal position the photographic paper will retain its original dimensions almost unchanged and readings will not be materially affected for this reason, provided the scale is taken sufficiently ample (e.g. 1 °C corresponding to 1cm). Copies should, however, always be compared with the original graph before final preparation.

3. READING OF THE CORRECTION GRAPH - A non-transparent ruler with the same scale as the graph is placed parallelly to the abscissa and adjusted in such a way that its upper edge intersects the ordinate at the given value T and that this value on the scale of the ruler lies on the curve of index correction. The given value of t is read on the ruler and the total correction $I + \Delta T$ is taken out on the graph below this point. On Fig. 3 the ruler is adjusted to $T = 12.95$; $t = 23.6$.

When correcting an unprotected thermometer the ruler intersects the ordinate at T_u, while T_w lies on the curve of index correction.

4. CONCLUDING REMARKS - As the mathematical expressions on which the graph is based are always a very close approximation to the exact equations, the accuracy of corrections depends only upon the scale and exactness of the master-graph and its faithful reproduction as well. Further, index correction can be added according

to the indications given above and thermometer readings applied directly without sacrificing accuracy although the index correction may be as high as 1.0 °C.

The method of constructing a master-graph from which individual graphs can be prepared, adapted from Theisen (1947), makes preparation of individual graphs very easy if the necessary photo--reproduction equipment is available.

The correction graph has been in use for some time at this Institute and has been found to be simple and easy to read.

RESUMO

As fórmulas para a correção de termômetros reversiveis, protegidos e desprotegidos, apresentadas nêste trabalho, são derivadas diretamente a partir da equação exata da expansão térmica. Duas dessas fórmulas, válidas para termômetros protegidos ou desprotegidos respectivamente, adaptam-se particularmente bem ao cálculo de tabelas que podem servir para a confecção de gráficos de correção. Destas pode-se obter a correção total com grande precisão partindo diretamente das leituras do termômetro usado, mesmo quando se tratar de termômetros cujo índice de correção seja igual a 1.0 °C.

Um gráfico de correção onde êsses resultados são aproveitados é descrito a seguir.

O método consiste em construir um gráfico matriz, um para termômetros protegidos, outro para os desprotegidos. Dêsses são tirados gráficos, reproduzindo-se fotogràficamente a porção desejada do gráfico matriz, para cada termômetro individualmente.

REFERENCES

HANSEN, W.
1934. Bemerkungen zu den Korrektionsformel für das Tiefsee-Umkippthermometer. Ann.Hydr.Marit.Met. Berlin, Jg. 62, p. 145-147.

THEISEN, E.
1947. Correction of temperatures and a handy way of making correction charts for reversing thermometers. Rep.Norweg.Fish.Mar. Invest. Bergen, Vol. 8, nº 9.

SVERDRUP, H.U.
1947. Note on the correction of reversing thermometers. Jour.Mar. Res. New Haven, Vol. 6, nº 2.

LaFOND, E.C.
1951. Processing oceanographic data. H.O.Publ. nº 614, Washington D.C.

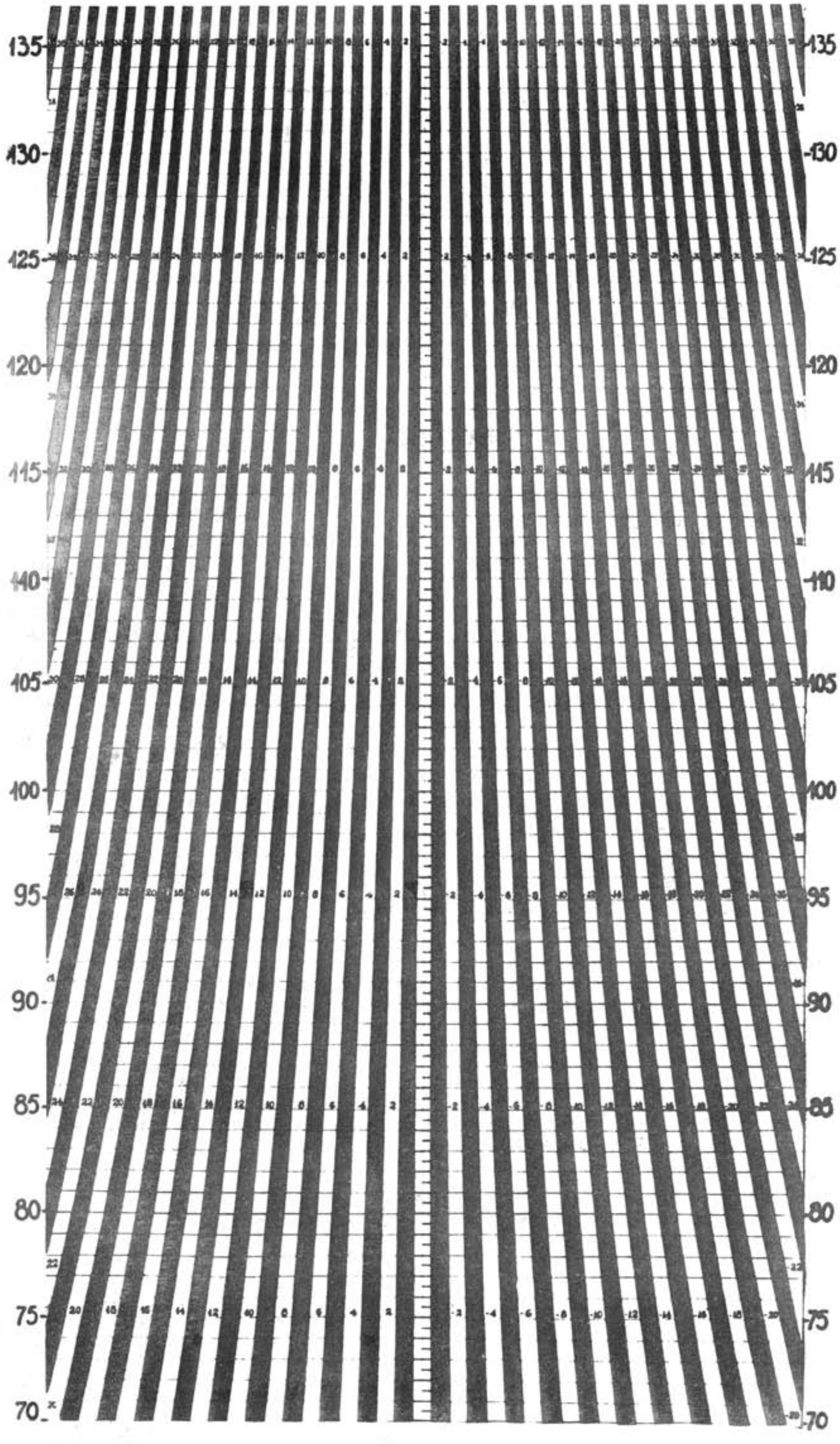

Fig. 2

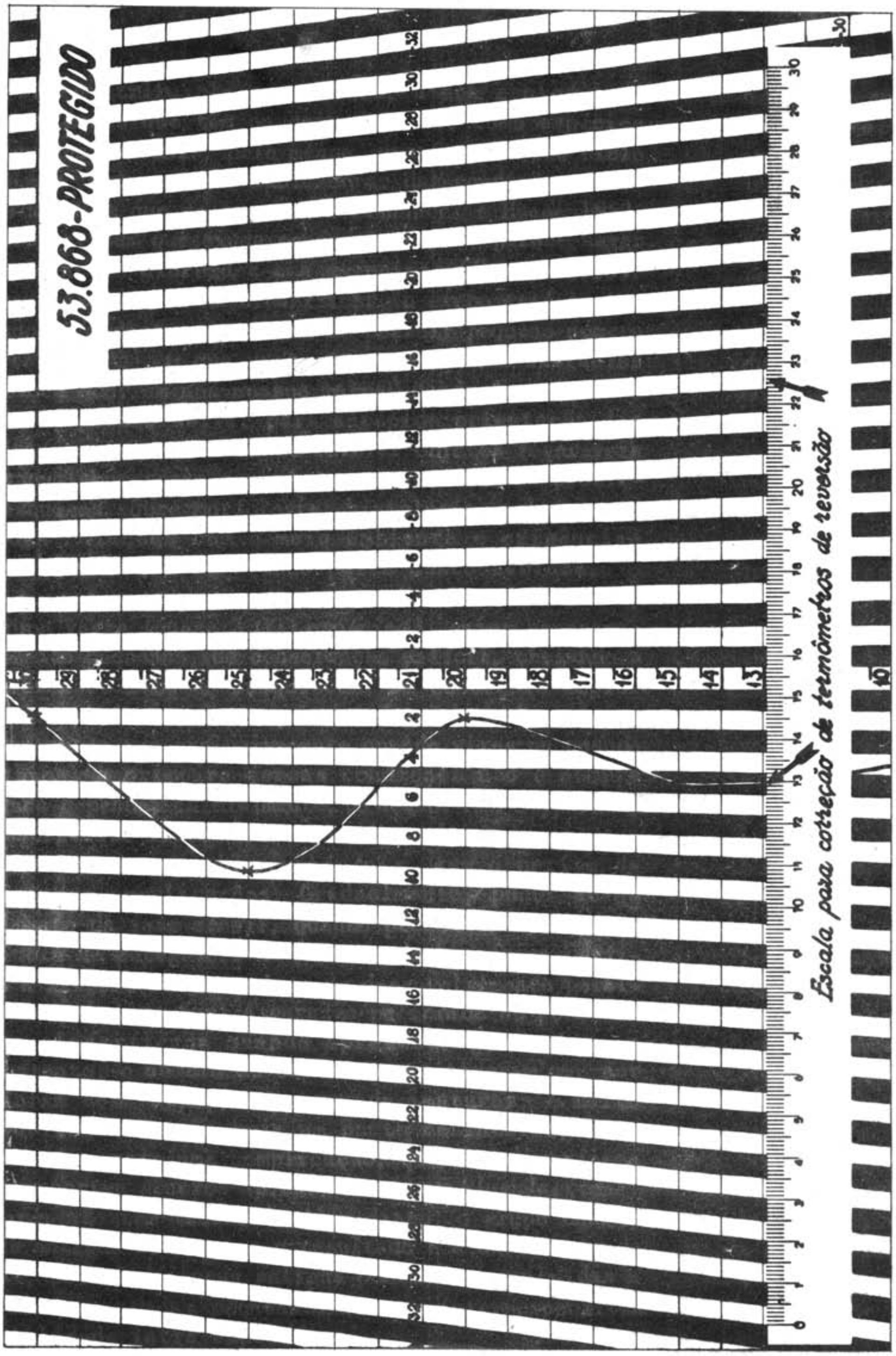
53.868-PROTEGIDO
Escala para correção de termômetros de reversão

Fig. 8