Artigos

Survival of technologies: an evolutionary game approach*

Eleutério F. S. Prado[§]
Décio K. Kadota[§]
Jorge E. de C. Soromenho[§]

RESUMO

Neste artigo, modelamos a adoção de tecnologias como um jogo evolucionário assimétrico baseado em encontros aleatórios bilaterais envolvendo duas populações, firmas e consumidores. Inicialmente, consideramos externalidades do lado da oferta e obtemos os resultados usuais da literatura recente sobre o tema: dependência de trajetória, "lock-in" e possibilidade de seleção de tecnologias inferiores. Em seguida, introduzimos externalidades no lado da demanda, as quais no artigo clássico de Leibenstein resultam nos efeitos cumulativo de consumo (bandwagon effect) e diferenciação de consumo (snob effect). Por último, examinamos as interações entre a oferta e a demanda.

Palavras-chave: competição evolucionária, racionalidade limitada, efeito cumulativo de consumo, efeito diferenciação de consumo e dependência de trajetória.

ABSTRACT

This paper models technology adoption as an evolutionary and asymmetric game based on a pairwise contest involving two populations, firms and consumers. First, externalities are considered only on the supply side, leading to the usual results found in the recent economic literature on the subject: path dependence, lock-in, and the possibility of selecting inferior technology. Next externalities are introduced on the demand side, which in Leibenstein's classic paper leads to bandwagon and snob effects, and interactions between supply and demand are examined.

Key words: evolutionary competition, bounded rationality, bandwagon effect, snob effect, path dependence.

JEL classification: L1, O3.

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§ Professores da FEA-USP.

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I Introduction

The survival of products or product technologies can be investigated using the classic analytical approach, which focuses on the demand side, as proposed by Leibenstein in 1950. When two products that meet the same consumer need compete in the marketplace, if the bandwagon effect prevails only one of them will tend to survive. If the snob effect prevails, however, both products will tend to survive and share the market between them. Positive network externalities predominate in the former case, negative network externalities in the latter.

The reasons for product technology survival can also be investigated using production theory, which emphasizes the supply side. Traditional production theory assumes non-increasing returns of scale and this leads to a predictable equilibrium for prices and market shares. Besides, this equilibrium is the most efficient allocation of resources. However, if increasing returns prevail there may be multiple equilibria and hence outcomes that cannot be analytically predicted.[1] Using stochastic processes to model technology adoption with increasing economies of scale, Arthur (1989) suggests that equilibrium is arrived at through an accumulation of minor causes characterized by path dependence and by the possibility of lock-in.

An important factor in technology adoption processes is whether positive or negative externalities are present, since these may drastically change the outcome. The above-mentioned studies take externalities into account to some extent on both the supply and demand side but do not treat these two influences simultaneously in a unified analytical framework.

Moreover, Leibenstein's classic paper remains within the confines of static analysis, as is well known. The author himself acknowledges, nonetheless, that the problems raised by sequential processes of technology and product adoption require a dynamic approach. The purpose of this study is to take a step toward the construction of a more general analytical framework, presenting the question as one that depends intrinsically on various different dynamics arising on both the supply and demand sides.

Thus product diffusion and technology adoption are modeled here as an evolutionary and asymmetric game based on a pairwise contest involving two populations, firms and consumers, characterized in both cases by bounded rationality. The firms, whose costs depend on the market share captured by the technology they employ, among other factors, aim to increase their

1 The technology adoption process can also be investigated empirically, as demonstrated in Chow (1967).

profit. Consumers aim to increase the surplus obtained in market transactions. Firms set prices. Interaction between firms and consumers takes place in a dynamic context in which time is an arrow and equilibrium is merely a possible event at the "end of history"

II Evolutionary game

Let the evolutionary game be defined in the context of a market in which two goods provide the same type of service but can be differentiated by consumers according to the technology used to produce them, e.g. videocassette recorders with VHS and Betamax systems. Firms may opt for strategy s_1 or s_2, each of which corresponds to one of these alternative technologies. Because the total number of firms is considered constant, it is admitted for the sake of normalization that nn_1 and $\nu_2 = 1 - \nu_1$ represent the numbers of firms that adopt each of the two technologies. Consumers opt for strategy r_1 or r_2 according whether they choose goods produced using technology 1 or goods produced using technology 2. Let μ_1 and $\mu_2 = 1 - \mu_1$ be the proportions of consumers opting for each alternative.

In a pairwise contest, firms and consumers are randomly removed from their respective populations, one by one, to take part in rounds of the game. The resulting encounters may or may not be effective from the economic standpoint. Effective transactions occur only in the case of strategy combinations (s_1, r_1) or (s_2, r_2) Suppose, however, that the product is a consumer durable. If a unit is not sold during a given round, it is held in inventory at no cost so as to participate in the next round. Thus in the case of strategy combinations (s_1, r_2) or (s_2, r_1), producers and consumers merely take payoffs equal to zero.

Producers review their strategies using a process of imitation dynamics in a context of bounded rationality. Strategy reviews are formalized with a replicator dynamics similar, but not identical, to Nachbar's interpretation (1990).

According to Nachbar's interpretation, each player compares their payoff with that of another player randomly chosen from the same population. A strategy switch is possible only if the comparison is made with a player who adopts the alternative strategy. It is assumed that there is a switching cost in accordance with a uniform distribution of probabilities. Given the difference in payoffs resulting from each of the two strategies, the proportion of players who effectively switch to the higher payoff strategy will depend on the probability that the switching cost is lower than or equal to this difference.

Let the payoffs accruing to players in the two populations be $U(s_i, \nu_1; r_j)$ and $U(r_j, \nu_1; s_i)$, where $i, j = 1, 2$. Then, we have the following payoff matrix:

		Producer	
		s_1	s_2
Consumer	r_1	$(U(r_1, \nu_1, s_1); U(s_1, \nu_1; r_1))$	$(U(r_1, \nu_1; s_2) = 0; U(s_2, \nu_1; r_1) = 0)$
	r_2	$(U(r_2, \nu_1, s_1) = 0; U(s_1, \nu_1; r_2) = 0)$	$(U(r_2, \nu_1, s_2); U(s_2, \nu_1; r_2))$

Given that these payoffs are nil when $i \neq j$, in Nachbar's interpretation the replicator dynamics for firms is as follows:

$$\dot{\nu}_1 = \nu_1(1-\nu_1)\left[U(s_1, \nu_1; r_1)\mu_1 - (1-\mu_1)U(s_2, \nu_1; r_2)\right] \tag{1}$$

Consider the imitation process that makes possible equation (1). If a producer does not sell his product in a given meeting and if he compares his payoff with that of another producer that has employed the alternative strategy and has sold his product, he switches to the successful strategy. In a certain way, it is possible to say that this player can interpret the sale failure in that given meeting as an indication that there is an offer surplus in the market. This behavior is compatible with the hypothesis that the player does not know the market conditions fully and that he follows rules (in this case, he follows a very simple imitation rule).

However, even when the two technologies coexist in the market, it is always possible that a firm does not succeed in selling its product, even if its technology is more profitable than the other. In others words, a sale failure in a given meeting does not imply that there is an offer surplus in the market. If bounded rationality prevails, it is possible to say that the player should orient himself by signals that synthesize the global conditions of the market (as it was affirmed by Hayek in The Use of Knowledge in Society). Considering the payoffs obtained by two different players that succeed in selling their products, the difference between them furnishes a signal that satisfies this last requirement. As it will be seen by means of equation (4), this difference expresses the two technologies prices variations that depend on the global offer or demand surplus.

In this way, an alternative replicator dynamics can be obtained straightforward supposing that the players always compare the payoffs obtained by two successful players that had adopted different strategies. In this case, even if the players orient themselves by means of rules in a bounded rationality context, they interpret market signals not as in the Nachbar's replicator dynamics. Now, the sale failure is not considered an indication of the global state of the business. Instead, the players gauge the market conditions by looking at the successful sales profitability. From an individual point of view - and in this paper context -, this kind of behavior is more plausible than of the one implicit in Nachbar's imitation process. Then, we have the following equation:

$$\dot{\nu}_1 = \nu_1(1-\nu_1)\left[U(s_1,\nu_1;r_1) - U(s_2,\nu_1;r_2)\right] \tag{2}$$

Consumers review their strategies in accordance with a process that does not involve interpersonal comparisons of satisfaction and is termed "satisficing dynamics" in the literature. (Vega-Redondo, 1996, p. 91) Individuals learn not from each other but from themselves, by comparing the average payoff obtained from the chosen strategy with a satisfaction benchmark. If the average exceeds the benchmark, they do not switch strategies; if the average falls short of the benchmark, the probability of a strategy switch depends on the frequency with which the new strategy is adopted. Based on this assumption we can obtain a replicator dynamics for consumers that is analogous to the previous one:

$$\dot{\mu}_1 = \mu_1(1-\mu_1)\left[U(r_1,\mu_1;s_1) - U(r_2,\mu_1;s_2)\right] \tag{3}$$

Lastly, assume prices change over time in accordance with excess demand:

$$\dot{\pi}_i = \lambda[\mu_i - \nu_i], \quad i = 1,2 \tag{4}$$

with $\lambda > 0$. If we normalize prices using $\pi_1 + \pi_2 = 1$, we can consider only the price equation for good 1.

III Demand side

Let the demanders' payoffs be the consumers' surpluses, which are measured by the difference between the price they are prepared to pay for the good, represented by P_i, and the market price, π_i. In each round the consumer may buy at most one unit of each good. Three cases can be distinguished in an analysis of demand functions.

(a) In the first case, consumer preferences do not depend on the consumption decisions of other players. Inverse demand for goods 1 and 2 is therefore independent from μ_1, and can be expressed using constant functions:

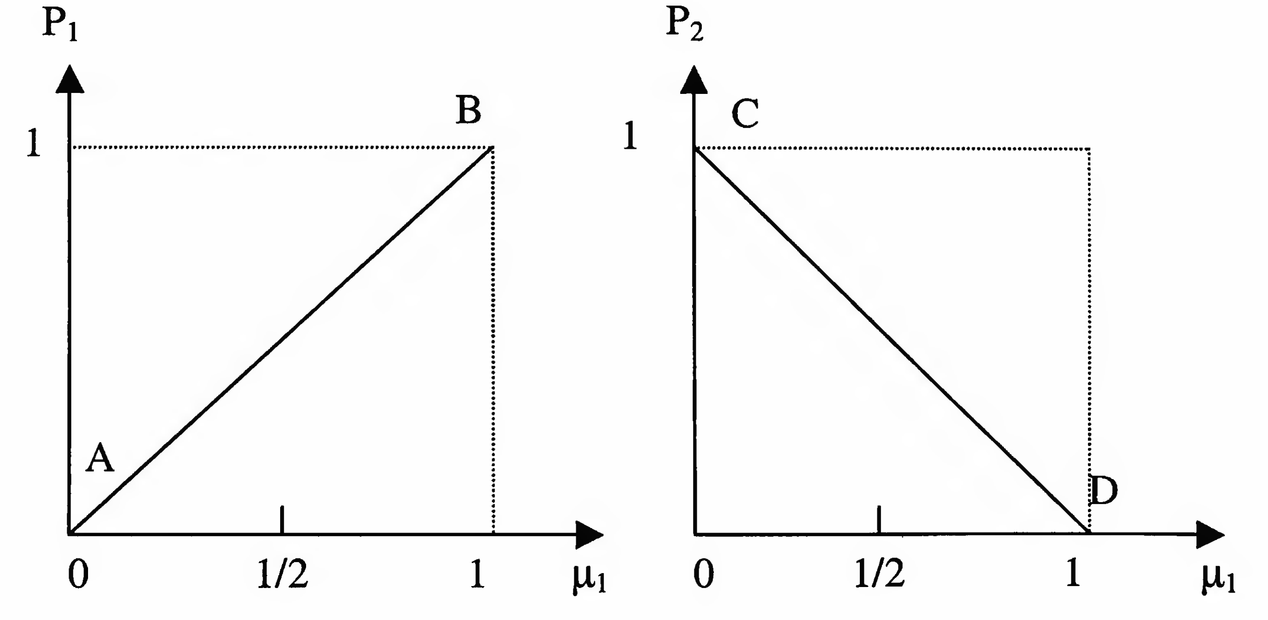
$$P_i = 1 \text{ and } U(r_i, \mu_i; s_i) = 1 - \pi_i, \text{ for } i = 1, 2.$$

(b) The second case considers a situation in which consumers prefer what the majority prefers. Although the two products provide the same type of service, there is a positive network externality which gives rise to a bandwagon effect. Thus:

$$P_i = \mu_i \text{ and } U(r_i, \mu_i; s_i) = \mu_i - \pi_i, \text{ for } i = 1, 2.$$

In Figure I, the straight-line segments AB and CD show the two characteristic behaviors of this payoff structure. If the figures were superimposed, they would intercept at the point where $\mu_1 = 0.5$ and $P_i = 1/2$, enabling the following properties to be seen more clearly· if $0 \leq \mu_1 < 0.5$, then $U(r_1) < U(r_2)$ and as consumers tend to acquire more of good 2, μ_1 decreases; if $0.5 < \mu_1 \leq 1$, then $U(r_1) > U(r_2)$ and μ_1 increases. In both cases the solution tends toward the extremes at which only one product dominates the market.

Figure I

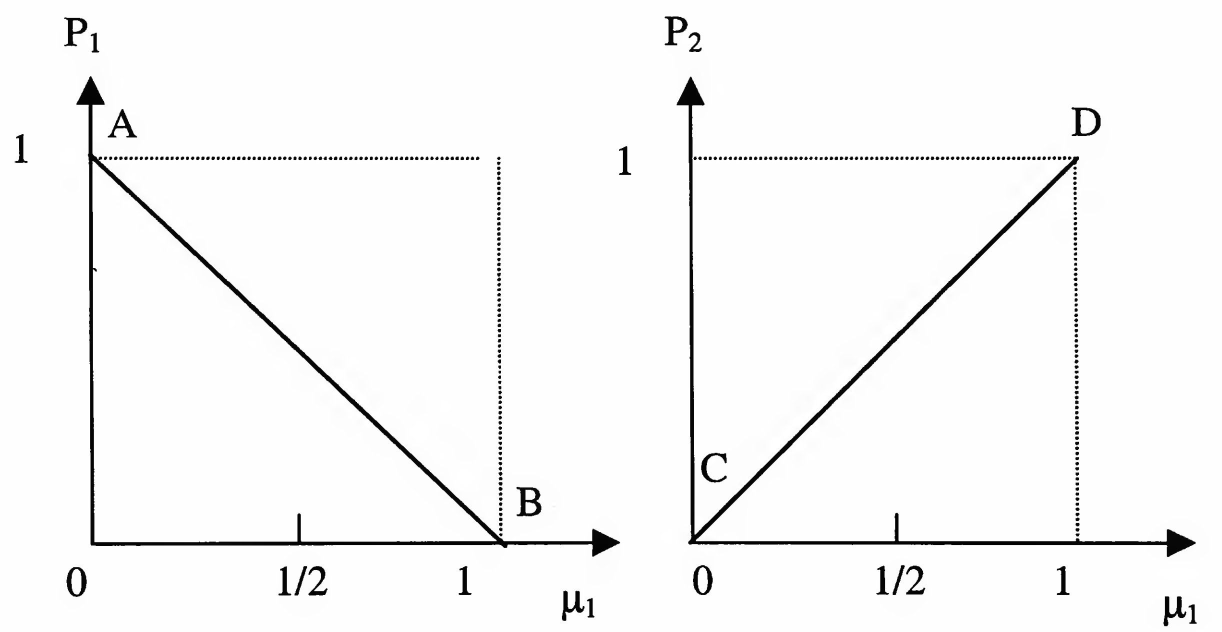


(c) In the latter case the effect is inverse: The smaller the number of consumers who buy the product, the larger the player's surplus at the same price. In other words, there is a negative network externality which gives rise to a snob effect. Thus

$$P_i = (1-\mu_i) \text{ and } U(r_i, \mu_1; s_i) = (1-\mu_i) - \pi_i, \text{ for } i = 1, 2.$$

The characteristic behaviors of this payoff structure, i.e. inverse demand for goods 1 and 2, can as before be represented graphically using straight-line segments AB and CD (see Figure II). If everyone is consuming good 2, for example, so that $\mu_1 = 0$, this is not an equilibrium situation since consumers will prefer to increase their demand for good 1 and as they do so μ_1 increases. Conversely, if everyone is consuming good 1 and demand for good 2 gradually rises, the proportion μ_1 decreases. Thus consumers tend to move away from the two extremes and the two goods end up sharing the market between them.

Figure II



If we obtain the average payoffs and then use (3), we generate differential equations for these three types of preference: neutral, producing a bandwagon effect, and producing a snob effect:

$$\dot{\mu}_1 = \mu_1(1-\mu_1)(1-2\pi_1) \tag{5}$$

$$\dot{\mu}_1 = 2\,\mu_1(1-\mu_1)\,(\mu_1-\pi_1) \tag{6}$$

$$\dot{\mu}_1 = 2\,\mu_1(1-\mu_1)(1-\pi_1-\mu_1) \tag{7}$$

IV Supply side

Let the unit costs of the firms representing the two groups be defined by the following linear functions:

$$c_i = \alpha_i + \beta_i\, q_i + b_i \nu_i \qquad i = 1, 2.$$

Hence costs depend on production scales and external economies, represented by parameters β and b respectively. The following cases can therefore be distinguished. First, if b_i is equal to, greater than or smaller than zero, external economies are non-existent, negative or positive respectively. Second, if β is equal to, greater than or smaller than zero, technologies present constant, decreasing or increasing economies of scale. Considering the scenario characterized by pairwise contests, each firm will produce only one unit per round. Thus the discussion does not focus on changes in scale but on each firm's average cost depending on the technology used and the externalities. The cost functions can therefore be simplified by assuming that $\alpha_i + \beta_i\, q_i = a_i$

$$c_i = a_i + b_i \nu_i, \qquad i = 1, 2.$$

If payoffs to firms are defined as profit per unit produced, then:

$$U(s_1, \nu_1; r_1) = \pi_1 - a_1 - b_1 \nu_1;$$

$$U(s_2, \nu_1; r_2) = \pi_2 - a_2 - b_2(1-\nu_1).$$

Assume that $c_i \leq \max\{\pi_i\}$. If this condition is not satisfied there will be no price that produces positive payoff for firms. Because costs must be strictly positive and $0 \leq \pi_i \leq 1$, by normalization we have $0 < a_i \leq 1$ and $-1 < b_i < 1$.

Substituting those payoffs into (2), we can obtain the following differential equation to represent the replicator dynamics:

$$\dot{\nu}_1 = \nu_1(1-\nu_1)\left[2\pi_1 + b_2 + a_2 - a_1 - 1 - (b_1 + b_2)\nu_1\right] \tag{8}$$

If there are no externalities, $b_1 = b_2 = 0$, and the above equation is reduced as follows:

$$\dot{\nu}_1 = \nu_1(1-\nu_1)\left[(a_2 - a_1 + 2\pi_1 - 1)\right] \tag{9}$$

V Dynamic interactions

By combining the various assumptions regarding the payoffs to firms and consumers, we come to three distinct situations, each of which involves a system of three differential equations. Price equation (4) and equation (8) for the replicator dynamics for firms are common to all three systems. The only difference is the equation for the consumer replicator dynamics: (5), (6) or (7).

Let Δ^3 be the domain of these systems:

$$\Delta^3 = \{(\nu_1, \mu_1, \pi_1) \mid 0 \le \nu_1, \mu_1 \le 1 \text{ and } 0 \le \pi_1 \le 1\}.$$

The stationary solution to equation (4) requires that ν_1 be equal to μ_1. Hence it is possible to analyze the stationary solutions to the systems in the following set:

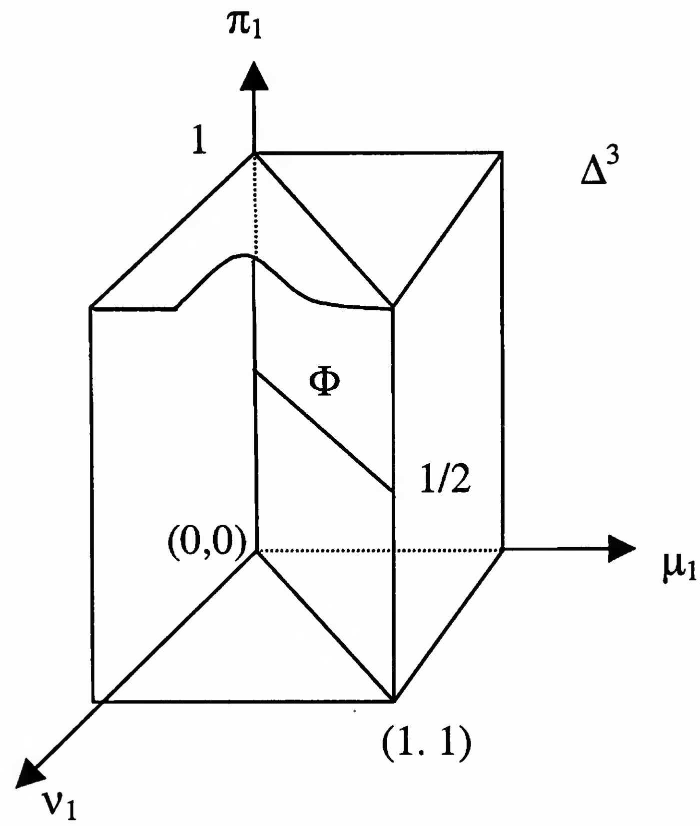
$$\Phi = \{(\nu_1, \mu_1, \pi_1) \mid 0 \le \nu_1 = \mu_1 \le 1 \text{ and } 0 \le \pi_1 \le 1\}.$$

As is evident, Δ^3 turns out to be a parallelepiped in R_3 (base 1 x 1 and height 1), while Φ turns out to be a rectangle in one of the planes that divide Δ^3 diagonally into equal parts. Both are shown in Figure III.

Now consider first of all the cases in which there are no externalities on the demand side. The system to be considered is formed by equations (4), (5), and (8) or (9).

Clearly, $\nu_1 = \mu_1 = 0$ and $\nu_1 = \mu_1 = 1$ constitute equilibria whatever the values of parameters a_i and b_i $(i = 1, 2)$ The possibility of mixed-strategy equilibrium $0 < \nu_1 = \mu_1 < 1$, depends on the values of both cost function parameters.

Figure III



Consider first of all a situation in which there are no externalities $(b_1 = b_2 = 0)$ and firms have the same scale-related costs ($a_1 = a_2$). A mixed-strategy equilibrium is possible: equations (5) and (9) require only that $\pi_1 = 1/2$.[2] Hence it can be concluded that all points in sets

$$E_1 = \{(\nu_1, \mu_1, \pi_1) \mid \nu_1 = \mu_1 = 0 \text{ and } 0 \leq \pi_1 \leq 1\},$$

$$E_2 = \{(\nu_1, \mu_1, \pi_1) \mid \nu_1 = \mu_1 = 1 \text{ and } 0 \leq \pi_1 \leq 1\},$$

and

$$E_3 = \{(\nu_1, \mu_1, \pi_1) \mid \nu_1 = \mu_1 \text{ and } \pi_1 = 1/2\}$$

2 In the case where $a_1 \neq a_2$, (5) requires $\pi_1 = M/2$, but (8) and (9) require $\pi_1 = (M + a_1 - a_2)/2$, so that these two conditions cannot be satisfied at the same time. Note that if technology 1 is less efficient, the difference $a_1 - a_2$ will be positive, so that π_1 would have to be greater than π_2 for both technologies to survive.

are stationary. It is easy to see that in this case the stationary points in the complete system form an H in plane Φ.

In sum, when there are no external economies and firms have the same scale-related costs, the market can be shared by the two technologies in any proportion, as one would expect. If there is a difference in scale-related costs, because the demand side requires $\pi_1 = \pi_2$ as a condition for stationarity, competition between firms will tend to evict the less efficient technology from the market and select only one in a path-independent manner.

Now admit the presence of externalities $(b_1 \text{ and } b_2 \neq 0)$, be they positive or negative. We obtain the following mixed-strategy equilibrium:

$$\pi_1 = \frac{1}{2} \text{ and } \nu_1 = \mu_1 = \frac{b_2 - a}{b_1 + b_2} \tag{10}$$

where $a = a_1 - a_2$, provided $b_1 + b_2 \neq 0$ and $0 < (b_2 - a)/(b_1 + b_2) < 1$.

Among the various possible combinations of the values for the parameters that satisfy the above condition, it is convenient to highlight those which correspond to positive or negative externalities for both technologies. If the external economies are negative $(b_1 \text{ and } b_2 > 0)$, there will be the possibility of a stationary equilibrium with survival of technologies having different scale-related costs that satisfy $a - b_2 < 0$ and $a + b_1 > 0$. The stability analyses (see Appendix) show that this equilibrium turns out to be stable.

If the external economies are positive $(b_1 \text{ and } b_2 < 0)$, any mixed-strategy equilibrium is unstable, so that stable economies exist only when $\nu_1 = 1$ or $\nu_1 = 0$. Thus the final equilibrium depends on the initial conditions and is path-dependent.

In sum, when preferences are neutral between 1 and 2, the outcome depends on scale-related costs and external economies. If there are no external economies $(b_1 = b_2)$, market equilibrium may occur in any fraction of the market, as shown earlier. If external economies are negative, the technologies may share the market in a proportion that depends on their relative efficiencies; if they are positive, only one of the technologies may survive, depending on the path taken by the economy. The above analysis shows that evolutionary game theory is an adequate framework to reproduce some of the results found in the recent economic literature on survival of technologies, such as path dependence, lock-in, selection of inferior technology etc.

Now consider a situation in which there is a bandwagon effect. In this case equations (4), (6) and (8) form the system. Examination of differential equation (6) shows that it presents three stationary solutions: $\mu_1 = 0$, $\mu_1 = 1$, and $\mu_1 = \pi_1$. Demand-side dynamics restricts the possible stationary points to points that belong to sets E_1, E_2 and

$$E_4 = \{(\nu_1, \mu_1, \pi_1) \mid \nu_1 = \mu_1 = \pi_1 \text{ and } 0 < \pi_1 < 1\}.$$

Note that these three sets form the figure $|/|$ in Φ of Δ^3

All points in sets E_1 e E_2 satisfy equation (8), which portrays the supply-side dynamics, and equation (4). However, not all points in set E_4 are stationary solutions. Whether or not there are external economies, the mixed-strategy equilibrium in E_4 is unique:

$$\pi_1 = \mu_1 = \nu_1 = \frac{b_2 - a - 1}{b_1 + b_2 - 2} \tag{11}$$

provided $b_1 + b_2 \neq 2$ and $0 < (b_2 - a - 1)/(b_1 + b_2 - 2) < 1$. Thus in any possible case the figure in Φ formed by the stationary points in the system is $|\ \ |$.

Stability analysis shows that this mixed-strategy equilibrium is always unstable (see Appendix). For this very reason, Leibenstein's result invariably comes out. This is the case even if external economies are negative. The bandwagon effect always prevails and only one technology survives.

Lastly, consider the presence of the snob effect. In this case the system is made up of equations (7), (4) and (8). Analysis of differential equation (7) shows that the stationary solutions are $\mu_1 = 0$, $\mu_1 = 1$ and $\mu_1 = 1 - \pi_1$. As before, demand-side dynamics restricts the possible existence of stationary points to points belonging to sets E_1, E_2 and

$$E_5 = \{(\nu_1, \mu_1, \pi_1) \mid \nu_1 = \mu_1 = 1 - \pi_1 \text{ and } 0 \leq \pi_1 \leq 1\}.$$

Note that these three sets form the figure N in Φ.

Equation (8) presents as a solution, besides $\nu_1 = 0$ and $\nu_1 = 1$, only one isolated point belonging to E_5. Thus we have a mixed-strategy equilibrium:

$$\pi_1 = 1 - \nu_1 \text{ and } \mu_1 = \nu_1 = \frac{b_2 - a + 1}{b_1 + b_2 + 2}. \tag{12}$$

provided $b_1 + b_2 \neq -2$ and $0 < (b_2 - a + 1)/(b_1 + b_2 + 2) < 1$. The stationary points in the system again form the figure | | in Φ.

In this case, stability analysis indicates that demand-side dynamics favors a solution in which both products or product technologies survive in the market. However, in contrast with the previous case, supply-side externalities can modify this expected result. In the appendix we show that a mixed-strategy equilibrium may be unstable if the production externalities are positive. More precisely, if $b_1 + b_2 < 0$, instability may occur depending on parameter ll which defines the process of price adjustment.

VI Conclusions

This paper analyzes technology survival within an evolutionary framework. It assumes that agents have bounded rationality and that as time goes by they tend to choose between strategies depending on the relative rewards. Initially, assuming an absence of network externalities, the results were found to be in accordance with the recent economic literature on the subject, including Arthur's papers among others. If the supply-side externalities are positive, one technology survives and the equilibrium is path-dependent; moreover, the survival process may select the less efficient technology. If the external economies are negative, both technologies may survive and share the market independently of the initial conditions.

The next step was to investigate the survival of technologies when there are both network externalities and external economies. The demand-side externalities produce bandwagon or snob effects, which in themselves make a mixed-strategy equilibrium unstable or stable respectively. When there is a bandwagon effect, no matter which external economy prevails on the supply side, the market tends to adopt only one technology and the equilibrium will be path-dependent. Demand-side externalities produce instability that will prevail in the market. The conclusion was similar for cases in which there is a snob effect, but the result was less definitive. If external economies are negative, the effects of supply-side and demand-side externalities are convergent and both technologies will survive. However, if external economies are positive, depending on the values of the parameters a result like the former may not come out, and only one technology will survive in spite of the snob effect.

Finally, it is important to note that although these conclusions were obtained in a dynamic framework, they are very similar to the findings obtained by Leibenstein in a static framework.

Appendix: stability analysis

A rest point is asymptotically stable if all eigenvalues of the Jacobian matrix have negative real parts and unstable if any eigenvalue has a positive real part. Let $\theta^3 + c_1\theta^2 + c_2\theta + c_3 = 0$ be the characteristic polynomial of a system of three differential equations. The Routh-Hurwitz necessary and sufficient conditions for stability are: $c_1 > 0$, $c_3 > 0$, and $c_1c_2 - c_3 > 0$

First system: no network externalities.

Evaluating at equilibrium (10), the Jacobian matrix of (4), (5) and (8) is given by

$$J = \begin{pmatrix} 0 & 0 & 2\gamma(b_1+b_2)^{-1} \\ 0 & \gamma & -2\gamma(b_1+b_2)^{-1} \\ \lambda & -\lambda & 0 \end{pmatrix}$$

where

$$\gamma = \frac{(a+b_1)(a-b_2)}{b_1+b_2},$$

and the characteristic equation is

$$\theta^3 - \gamma\,\theta^2 - \frac{4\gamma\lambda}{b_1+b_2}\theta + \frac{2\gamma^2\lambda}{b_1+b_2} = 0$$

If the mixed-strategy equilibrium exists and the external economies are negative $b_1, b_2 > 0$, $a - b_2 < 0$ and $a + b_1 > 0$ So, $\gamma < 0$ and $c_1 = -\gamma > 0$; $c_3 = 2\gamma^2\lambda/(b_1 + b_2) > 0$; and $c_1c_2 - c_3 = 2\gamma^2\lambda/(b_1 + b_2) > 0$. Thus the Routh-Hurwitz conditions are satisfied. If external economies are positive, $b_1, b_2 < 0$ and $c_3 = 2\gamma^2\lambda/(b_1 + b_2) < 0$ Thus the equilibrium is unstable.

Second system: positive network externalities (bandwagon effect).

Evaluating at equilibrium (11), the Jacobian matrix of (4), (6) and (8) is given by

$$J = \begin{pmatrix} -2\alpha & 0 & 2\alpha \\ 0 & (b_1 + b_2)\alpha & -2\alpha \\ \lambda & -\lambda & 0 \end{pmatrix}$$

where

$$\alpha = \frac{(1 + a - b_2)(a + b_1 - 1)}{(b_1 + b_2 - 2)^2} < 0,$$

and the characteristic equation is

$$\theta^3 - (b_1 + b_2 - 2)\alpha\,\theta^2 - 2\alpha\left[(b_1 + b_2)\alpha + 2\lambda\right]\theta + 2(b_1 + b_2 - 2)\alpha^2\lambda = 0$$

As, by assumption , we have Hence the equilibrium is necessarily unstable.

Third system: negative network externalities (snob effect).

Evaluating at equilibrium (12), the Jacobian matrix of (4), (7) and (8) is given by

$$J = \begin{pmatrix} 2\beta & 0 & 2\beta \\ 0 & (b_1 + b_2)\beta & -2\beta \\ \lambda & -\lambda & 0 \end{pmatrix}$$

where

$$\beta = \frac{(1+a+b_1)(a-b_2-1)}{(b_1+b_2+2)^2} < 0 ,$$

and the characteristic equation is

$$\theta^3 - (b_1+b_2+2)\beta\,\theta^2 + 2\beta\left[(b_1+b_2)\beta - 2\lambda\right]\theta + 2(b_1+b_2+2)\beta^2\lambda = 0$$

As $b_1+b_2 > -2,\ c_1 > 0,\ c_3 > 0$. The third condition, $c_1c_2 - c_3 > 0$, implies $\lambda > (b_1+b_2)\beta$ As $\beta < 0$, if $0 \le b_1+b_2 < 2$, for $\lambda > 0$ any the equilibrium is stable. On the other hand, for the open set $-2 < b_1+b_2 < 0$, we can define $b_1+b_2 = -2+\varepsilon, 0<\varepsilon<2$, and

$$\phi[\varepsilon; b_2] := \beta(b_1+b_2) = \frac{(a-1-b_2)(a-1-b_2+\varepsilon)(\varepsilon-2)}{\varepsilon^2}$$

The function ϕ is continuous, monotonically decreasing in ε, with $\lim_{\varepsilon\to 0}\phi = \infty$ and $\lim_{\varepsilon\to 2}\phi = 0$ Therefore for any $\lambda > 0$ there is a partition of $-2 < b_1+b_2 < 0$ into two subsets: one where the equilibrium is stable, and the other where it is unstable.

References

Arthur, W. B. Competing technologies, increasing returns, and lock-in by historical small events. *Economic Journal* 99, p. 116-31, 1989.

Chow, G. C. Technological change and the demand for computers. *American Economic Review* 57, p. 1117-30, 1967

Farrel, J., Shapiro, C. The dynamics of bandwagons. *In*: Friedman, J. W. (ed.), *Problems of coordination in economics*. Boston: Kluwer Academic Publishers, 1993.

Janssen, M., Jager, W. An integrated approach to simulating behavioural processes: a case study of the lock-in of consumption patterns. *Journal of Artificial Societies and Social Simulation* 2 http://www.soc. surrey.ac.uk/JASSS, 1999.

Leibenstein, H. Bandwagon, snob, and Veblen effects in the theory of consumers' demand. *Quarterly Journal of Economics* 64, p. 12-30, 1950.

Nachbar, J. H. Evolutionary selection in dynamic games. *International Journal of Game Theory* 19, p. 59-90, 1990.

Vega-Redondo, F. *Evolution, games and economic behaviour*. Oxford: Oxford University Press, 1996.