A Non-Linear Macrodynamics of Capital Accumulation, Distribution and Conflict Inflation

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RESUMO

É elaborado um modelo pós-keynesiano de acumulação de capital, distribuição e inflação por conflito em que o investimento depende não-linearmente da distribuição: para níveis baixos (elevados) de parcela salarial, o efeito sobre o investimento de uma maior parcela de lucros é negativo (positivo). Esta especificação parece conformar-se com a evidência empírica para a ascensão e queda da Idade do Ouro na maioria das economias avançadas. Com isso, a economia estará em um regime de acumulação comandado pelos salários ou em um regime comandado pelos lucros dependendo da distribuição prevalecente. Dada essa não-linearidade, é delineada uma análise qualitativa de uma possível configuração marcada por equilíbrios múltiplos e flutuações endógenas e auto-sustentadas. Portanto, um aspecto inovador do modelo é que a mudança nos regimes de acumulação de capital e de crescimento não requer o alcance da plena utilização da capacidade.

PALAVRAS-CHAVE

acumulação de capital, distribuição, inflação por conflito

ABSTRACT

It is developed a post-keynesian model of capital accumulation, distribution and conflict inflation in which investment is non-linear in distribution: at low (high) levels of wage share, the effect of a higher profit share on investment is negative (positive). This specification seems to conform with the empirical evidence for the rise and fall of the Golden Age in most advanced economies. As it turns out, whether the economy follows a wage-led accumulation regime or a profit-led one depends on distribution. Given such non-linearity, it is developed a qualitative analysis of a possible configuration leading to multiple equilibria and endogenous, self-sustaining fluctuations. Hence, an innovative feature of the model is that it does not rely on full capacity utilization being reached for a change in the capital accumulation and growth regimes to take place.

KEY WORDS

capital accumulation, distribution, conflict inflation

JEL classification

E32; O10; O41

INTRODUCTION

This paper develops a post-keynesian dynamic macromodel of capital accumulation, distribution and conflict inflation. Firms' desired investment is made to be non-linear in distributive (wage and profit) shares, which implies that whether it will rise or fall in response to a change in distribution depends upon the level of distribution. It is assumed that firms' desired accumulation will be lower for both high and low levels of wage share, it being higher for intermediate levels of distribution. While at high (low) levels of wage (profit) share the impact of higher profitability on investment plans is positive, this impact will become negative at low (high) levels of wage (profit) share. The point of this formal specification is to introduce an analytically reasonable and empirically plausible non-linear feedback from distribution - which is jointly determined with actual capital accumulation in the class of models from which this one draws mostly - to firms' accumulation plans. Indeed, this specification seems to conform with the empirical evidence for the rise and fall of the Golden Age in most advanced economies. In turn, inflation dynamics is determined within a conflicting income claims framework, meaning that inflation occurs whenever workers and capitalists press claims in excess of available income.

As it turns out, whether the economy will follow a wage-led accumulation regime or a profit-led one depends on the prevailing distribution, with a similar dependence applying - along with the relative bargaining power of capitalists and workers and whether the growth rate of labour supply is exogenous or endogenous - to its dynamic stability properties. While stagnation or wage-led capacity utilization and growth will obtain only for sufficiently low levels of wage share, exhilarationism or profit-led capacity utilization and growth will obtain only for sufficiently low levels of profit share. Given such non-linearity, it is also developed a qualitative, phase-diagrammatic analysis of a possible configuration for the system leading to multiple equilibria and endogenous, self-sustaining fluctuations in its relevant variables.

Hence, an innovative feature of the model of this paper is that it does not rely on full capacity utilization being reached for a change in the capital accumulation and growth regimes to take place. Indeed, while the post keynesian approach to growth and distribution is mostly known for the models developed by N. Kaldor, J. Robinson and L. Pasinetti in the 1950s and 1960s, a distinction should be made between those earlier models and the newer ones developed independently by authors more closely associated with the Kalecki-Steindl tradition like R. Rowthorn and A. K. Dutt in the 1980s. While the older models implicitly assume that in the long run either full capacity is reached or capacity utilization is fixed at a given normal level, in the newer approach capacity utilization is endogenous in whatever run. Consequently, while in the older post-keynesian approach there is an inverse relation between the wage share and the rates of capital accumulation and growth, such relation is usually positive in the newer one.[1]

This paper is organized in the following way. Section 1 describes the structure of the model, whereas section s analyses its behaviour in the short run. The behaviour of the model in the long run is discussed in section 3, under the assumption that the labour supply growth is exogenous given. Section 4 analyses its dynamics under the assumption that the growth of labour supply is endogenous. While section 5 examines one possible long-run multiple equilibria dynamics leading to the emergence of cyclical behaviour, the final section presents a summary.

1 While there is no special connection between the old neoclassical model of growth developed by SOLOW (1956) and the determination of income distribution, the more recent neoclassical theoretical literature on such connection has expanded enormously. This literature provides an array of very different explanations for a positive correlation between income equality and growth, all of them quite different from the post-keynesian one suggested in this paper, though. To put it briefly, in this mainstream literature an increase in inequality is argued to lower growth by raising redistributive government expenditure and therefore distortionary taxation, by increasing sociopolitical instability, or by reducing private investment in human capital. Some surveys of this literature can be found in BENABOU (1996), PEROTTI (1996), AGHION, CAROLI & GARCIA-PEÑALOSA (1999) and LIMA (2000b).

1. THE STRUCTURE OF THE MODEL

The economy is a closed one and with no government activities, producing only one good for both investment and consumption. Two factors of production are used, capital and labour, the technology being one of fixed coefficients:

$$X = \min\ [Ku_k, L/a] \tag{1}$$

where X is the output level, K is the capital stock, L is the employment level, u_k is the level of technologically-full capacity utilization, while a stands for the labour-output ratio. These parameters are assumed to remain unchanged throughout, since we abstract from technological change.[2] The fixed-coefficients assumption can be justified by reference either to technological rigidities in factor substitution or to a choice of techniques independent of factor prices.

Production is carried out by oligopolistic, price-maker firms. At a point in time, prices are given, having resulted from past dynamics. Firms will produce according to demand, it being assumed that forthcoming demand is insufficient for them to produce at full capacity at the ongoing price.[3] Labour employment is determined by production:

$$L = aX \tag{2}$$

2 A post-keynesian dynamic macromodel of capital accumulation, growth and distribution in which labour-saving technological innovation plays a pivotal role is developed in LIMA (2000a). The rate of technological innovation is made to depend non-linearly on the degree of market concentration, thus incorporating a possibility discussed in the neo-schumpeterian literature on the double-sided relationship between industrial dynamics and technological change.

3 STEINDL (1952) claims that firms plan excess capacity so as to be ready for a sudden expansion of sales. First, the existence of fluctuations in demand means that the producer wants to be in a boom first, and not to leave the sales to new competitors who will press on her market when the boom is over. Second, it is not possible for the producer to expand her capacity step by step as her market grows because of the indivisibility and durability of the plant and equipment. Finally, there is the issue of entry deterrence: if prices are sufficiently high, entry of new competitors becomes feasible even where capital requirements are great; hence, the holding of excess capacity allows olipolistic firms to confront new entrants by suddenly raising supply and driving prices down. In the same vein, SPENCE (1977) and COWLING (1982) argue that the holding of excess capacity inhibits entry by making potential entrants unsure about post-entry profits.

meaning that no excess labour is employed by firms - we abstract from long-term labour contracts. Firms also make accumulation plans, the assumption being that they have a desired investment function of the form

$$g^d = \alpha + \beta u + \gamma\sigma - \delta\sigma^2 \tag{3}$$

where α, β, γ, δ with $\gamma \geq \delta$, are all positive parameters of the desired investment function, g^d, which is firms' desired accumulation as a ratio of the existing capital stock, σ is the share of wages in income, and $u = X / K$ is the actual rate of capacity utilization. Since we assume that the ratio of capacity output to the capital stock is constant, we can therefore identify capacity utilization with the actual output-capital ratio. Indeed, this identification is a useful simplification in that it keeps the dimensionality of the model low, since additional variables which as well determine productive capacity, such as labor force, are omitted.

We follow Rowthorn (1981) and Dutt (1984, 1990), who in turn follow Steindl (1952), in assuming that desired investment depends positively on capacity utilization due to accelerator-type effects.[4] However, while Rowthorn and Dutt follow Kalecki (1971) and Robinson (1956, 1962) and make desired investment to depend positively on the profit rate, we make it to depend on distributive shares instead.[5] Rather than following

4 As recalled above, STEINDL (1952) argues that firms aim at the preservation of a certain margin of excess capacity. In the event, therefore, that actual excess capacity falls below the desired one, firms will tend to speed up the pace of capital accumulation. Interestingly, in turn, variable capacity utilization has been claimed by proponents of the real business cycle approach to be a source of propagation of technology shocks. In BURNSIDE & EICHENBAUM (1996), for example, firms will choose to hoard capital to be ready to increase the effective stock of capital at once in response to shocks that raise the marginal product of capital. In the canonical real business model, firms would have to wait at least one period to raise the stock of capital, and to the extent that capital takes time to build, they would have to wait even longer. It should be stressed, however, that the supply-side nature of this approach makes its endogenous determination of capacity utilization quite distinct from the demand-driven one pursued in the model of this paper.

5 BHADURI & MARGLIN (1990) argues for a formulation of desired investment as a function of the profit share, rather than the profit rate, on the ground that this clearly separates the two influences at work whereas the rate of profit reflects the dual influences of profit share and capacity utilization. In their view, to use the rate of profit is tantamount to assume that a given rate of profit will produce the same level of investment as results from high capacity utilization and a low profit margin or from low capacity utilization and a high profit margin. That is, this specification is insensitive to the influence of the existing capacity utilization, e.g. it neglects the possibility that, despite a high profit margin, firms may not be willing to invest in additional capacity if massive excess capacity already exists.

Bhaduri & Marglin (1990) in making firms' desired investment to depend implicitly monotonically on the profit share, though, we make it to depend non-linearly on the wage share. More precisely, we assume that desired investment will be lower for both high and low levels of wage share, it being higher for intermediate levels of distributive shares. While at high (low) levels of wage (profit) share the impact of higher profitability on desired investment is positive, this impact will become negative at low (high) levels of wage (profit) share.[6]

Indeed, one can read some of the empirics of the rise and fall of the Golden Age in the advanced capitalist economies (e.g. GLYN *et al*., 1990; MARGLIN & BHADURI, 1990; BHASKAR & GLYN, 1991; GLYN, 1997) as showing that the level of distribution matters for the direction and intensity of the profitability effect on investment. In Bhaskar & Glyn (1991), for instance, it is shown that while profitability is important for most of the countries, there is no simple association between increasing profits and increasing investment. Albeit a few outlier countries, the average empirical evidence reported in Marglin & Bhaduri (1990) shows that the corporate business net profit share fell almost monotonically throughout the period, while the share of business fixed investment in output rose during the Golden Age and fell during the 1970s and early 1980s.

6 BHASKAR (1992) investigates the effect of a rise in the wage rate on investment when demand is uncertain and incentives to factor substitution arise from the existence of different vintages of capital. Investment is undertaken to economize on labour costs by replacing older equipment and to meet additional demand. Higher wages reduce the return to incremental investment in supply-constrained states by reducing the absolute profit margin, but raise it in demand-constrained states. It is shown that there is a critical level of the wage above which the probability of being supply-constrained becomes dominant, with investment rising (falling) in the wage below (above) this critical level: investment is increasing (decreasing) in the wage when the wage is low (high). For Bhaskar, this non-linearity may provide a partial explanation for the dubious empirical evidence on the relationship between profit margins and investment presented in MALINVAUD (1980) and BHASKAR & GLYN (1991). Before claiming that this formulation provides a microeconomic foundation for the investment function used here, though, one should bear in mind that our model does not rely on factor substitution or vintage effects. Besides, in our model a positive relation between profitability and investment does not depend on output being supply-constrained - which in the model by Bhaskar means that firms are producing at full capacity.

The economy is inhabited by two classes, capitalists and workers. Following the tradition of Marx, Kalecki (1971), Kaldor (1956), Robinson (1956, 1962), and Pasinetti (1962), we assume that these groups have different saving behaviour. Workers provide labour and earn only wage income, all spent in consumption. This assumption that workers as a class do no saving does not, of course, rule out the possibility that individual workers might save. What this view amounts to is the assumption that for workers as a class the saving of some of them is matched by the dissaving of others. Besides, workers are always in excess supply, the actual number of potential workers growing at the rate n. Capitalists receive profit income, which is the entire surplus over wages, and save all of it. The division of income is given by

$$X = (W / P)L + rK \tag{4}$$

where W is the money wage, P is the price level, and r is the profit rate (the flow of money profits divided by the value of capital stock at output price). From (2) and (4), labour share is

$$\sigma = Va \tag{5}$$

where $V = (W / P)$ stands for the real wage. The profit rate can then be expressed as

$$r = (1-\sigma)u = \pi u \tag{6}$$

where π is the share of profit in income.

The price level is given at a point in time, but over time it will rise whenever the desired markup of firms exceeds their actual markups. Now, to a higher desired markup by firms a lower implied wage share, σ_f, will correspond. Formally,

$$\hat{P} = \tau[\sigma - \sigma_f] \tag{7}$$

where $\hat{P}$ is the proportionate rate of change in price, $(dP/dt)(1/P)$, and $0<\tau\leq 1$ is the speed of adjustment. Hence, inflation dynamics is determined within a conflicting claims framework, inflation resulting whenever income claims of classes exceed the available one. The price is determined à la Kalecki (1971), set by firms as a markup over prime costs:

$$P=(1+z)Wa \tag{8}$$

where z is the markup. Given labour productivity, $(1 / a)$, the markup is inversely related to the wage share, so that the gap between the desired and the actual markup can be measured by the gap between the actual and the firms' desired wage share. The desired markup by firms is taken to depend on the state of the goods market, it being assumed that a higher level of capacity utilization, which reflects more buoyant demand conditions, will induce firms to desire a higher profitability. Formally,

$$\sigma_f=\varphi-\theta u \tag{9}$$

where φ and θ are positive parameters.[7]

At a point in time the money wage is given, changing over time in line with the gap between the wage share desired by workers, σ_w, and the actual wage share:

$$\hat{W}=\mu[\sigma_w-\sigma] \tag{10}$$

7 WOOD (1975) and EICHNER (1976) argue that during expansions firms may want to invest more by generating higher internal savings and therefore desire a higher markup. ROWTHORN (1977) suggests that higher levels of capacity utilization allows firms to raise prices with less fear of being undercut by their competitors, who would gain little by undercutting due to higher capacity constraints. GORDON, WEISSKOPF & BOWLES (1984) argues that marked-up prices are inversely related to the perceived elasticity of demand, which is a negative function of the degree of industry concentration and of the fraction of the firm's potential competitors who are perceived to be quantity-constrained and hence not engaged in or responsive to price competition. The conclusion is that in the downturn the markup will fall because the general fall in capacity utilization gives rise to a smaller share of the firm's potential competitors being perceived to be operating under capacity constraints, and hence to an increase in the perceived elasticity of demand facing the firm.

where $\hat{W}$ is the proportionate rate of change in money wage, and $0 < \mu \leq 1$ is the speed of adjustment. The wage share desired by workers is assumed to depend on workers' bargaining power, which rises with the employment rate:

$$\sigma_w = \rho + \lambda e \tag{11}$$

where ρ and λ are positive parameters and e is the employment rate, L/N, which can linked to the state of the goods market in the following way

$$e = auk \tag{12}$$

where k stands for the ratio of capital stock to labour supply, $k = K / N$, with N being the supply of labour. This formal link between u and e is necessary because the fixed-coefficient nature of the technology implies that an increase in output in the short run will necessarily be accompanied by an increase in employment.

Since the model is demand-driven, the equality between investment and saving will be brought about by changes in output through changes in capacity utilization. Assuming that capital does not depreciate, g, the rate of capital accumulation, which is the growth rate for this one-good economy, is given by

$$g = r \tag{13}$$

given the assumptions that workers do not save and capitalists save all of their income.

2. *THE BEHAVIOUR OF THE MODEL IN THE SHORT RUN*

The short run is defined as a time frame in which the capital stock, K, the labour supply, N, the price level, P, and the money wage, W, can all be taken as given. The existence of excess capacity implies that output will adjust to remove any excess demand or supply, so that in short-run equilibrium, $g = g^d$. Substituting from (3), (6), and (13), we can solve for the equilibrium value of u, given σ and the other parameters:

$$u^* = \frac{\alpha + \gamma\sigma - \delta\sigma^2}{(1-\sigma) - \beta} \tag{14}$$

Meaningful values for the wage and profit shares are required, and a positive profit share is automatically ensured by $z > 0$. A positive wage share is ensured by $z < +\infty$, which we assume. As for stability, it is assumed a Keynesian short-run adjustment mechanism stating that output will change in proportion to the excess demand in the goods market, which means that the short-run equilibrium value for u will be stable provided the denominator of the expression in (14) is positive. This is ensured by the usual condition for macro stability that aggregate saving is more responsive than investment to changes in output (capacity utilization), which here implies that β has to be low enough, a condition we assume to be satisfied. Since α is positive, the numerator of the expression for u^* will be positive throughout its relevant domain, given by $0 < \sigma < 1$.

A natural issue to address regards the impact of changes in the wage share on capacity utilization. However, whether or not a higher wage share will increase the degree of capacity utilization is ambiguous, given that firms' desired investment is non-linear in distribution. This ambiguity is captured by the formal expression for $u_\sigma{}^*$:

$$u_\sigma{}^* = \frac{(\gamma - 2\delta\sigma) + u}{(1-\sigma) - \beta} \tag{15}$$

The restrictions already placed in the parameters ensure that u is positive throughout its (economically) meaningful domain, and substituting (14) into (15), we have

$$u_{\sigma}^{*} = \frac{\delta\sigma^{2} - 2\delta(1-\beta)\sigma + \gamma(1-\beta) + \alpha}{[(1-\sigma)-\beta]^{2}} \tag{16}$$

The numerator of this expression for u_{σ}^{*} is a concave-up parabola, and further suitable restrictions in the parameters will ensure that it will be zero at some $\sigma^{*} < \sigma^{+} < 1$ and at some $\sigma \geq \gamma / \delta$, with $\sigma^{*} = \gamma / 2\delta$ being the wage share at which desired accumulation, given u, is the highest. These will be the levels at which the expression for u_{σ}^{*} will change sign, and given that its numerator is a concave-up parabola, u_{σ}^{*} will be positive for $\sigma < \sigma^{+}$, and negative for $\sigma > \sigma^{+}$. While for low, intermediate-low and intermediate-high levels of wage share a higher wage share will increase capacity utilization, the reverse will happen for high levels of the wage share.

Following a terminology employed by Bhaduri and Marglin (1990), an stagnationist capacity utilization regime prevails at low, intermediate-low and intermediate-high levels of wage share, whereas an exhilarationist one prevails at high levels of wage share.[8] When $\sigma < \sigma^{*}$, a higher wage share will increase capacity utilization, as in the newer post keynesian model developed independently by Rowthorn (1981) and Dutt (1984, 1990). When $\sigma > \sigma^{*}$, the positive impact of a higher wage share on consumption is accompanied by a negative impact of lower profitability on desired investment. Since the intensity of this negative impact rises with the gap between σ^{*} and σ, the chances for $u_{\sigma}^{*} < 0$ will be the higher, the

8 As recalled on f. 5, BHADURI & MARGLIN (1990) argues that it is theoretically unsound to make investment to depend on capacity utilization and the rate of profit, for it is not certain that an increase in capacity utilization will induce additional investment when the profit rate is held constant - the reason being that in case capacity utilization increases while the rate of profit remains constant, it must be the case that the profit share falls. However, BHADURI & MARGLIN (1990) uses an investment function which is implicitly positively linear in the profit share, and the way it allows for the possibility that capacity utilization may rise (fall) when the profit shares rises is by suggesting that investment may be more (less) responsive than savings to changes in the profit share at high (low) levels of capacity utilization. Hence, while wage-led capacity utilization may be more likely at low levels of capacity utilization, profit-led capacity utilization may be more likely at high levels of capacity utilization. In the model of this paper, the desired investment function, by being explicitly non-linear in distributive shares, implies more naturally that whether investment or saving is more responsive to changes in profitability depends on the level of profitability itself.

closer σ gets to the upper bound for the wage share. Hence, suitable restrictions in the parameters will ensure that the numerator of (16) will be zero at $\sigma^* < \sigma^+ < 1$ and at some $\sigma \geq \gamma / \delta$, so that a rise in the wage share will rise capacity utilization for some intermediate-high levels of wage share as well. For high levels of wage share, the squeeze in profits brought about by a higher wage share will be strong enough to make for $u_\sigma^* < 0$.

Given our assumptions that workers do not save and capitalists save all of their income, the rates of profit and accumulation will be the same, and how these rates will respond to changes in distribution is likewise ambiguous. Given capacity utilization, a rise in the wage share will exert a downward pressure on the accumulation rate, while the non-linear nature of the desired investment function makes for the possibility that a higher wage share generates a rise in capacity utilization which may eventually more than compensate the accompanying fall in the profit share. For high levels of wage share ($\sigma > \sigma^+$), a redistribution towards workers will unambiguously slow down the rate of accumulation, the reason being that the resulting profit squeeze will reduce capacity utilization via its negative impact on desired investment. For the remainder of the relevant domain ($\sigma < \sigma^*$), a rise in the wage share, by raising capacity utilization, may eventually speed up the rate of accumulation, with the resulting profit-squeeze not being deleterious.

Hence, the non-linear investment function employed here allows us to derive some precise distributional conditions under which the relationship between distribution and accumulation is either positive or negative. All this ambiguity is captured by the formal expressions for g^* and g_σ^*, which, using equations (6), (13), (14), and (16), are given by

$$g^* = \frac{\delta\sigma^3 - (\gamma+\delta)\sigma^2 + (\gamma-\alpha)\sigma + \alpha}{(1-\sigma) - \beta} \tag{17}$$

$$g_{\sigma}{}^{*} = \frac{-2\delta\sigma^{3} + [3\delta(1-\beta) + (\gamma+\delta)]\sigma^{2} - 2(1-\beta)(\gamma+\delta)\sigma + (1-\beta)\gamma + \alpha\beta}{[(1-\sigma)-\beta]^{2}} \tag{18}$$

Further suitable restrictions in the parameters will ensure that the cubic expression in the numerator of (18) be zero at some $\sigma < 0$, at $\sigma = \sigma^*$, and at some $\sigma \geq \gamma / \delta$. For both low and intermediate-low levels of wage share ($\sigma < \sigma^*$) wage-led growth will prevail, whereas for intermediate-high and high levels of wage share ($\sigma > \sigma^*$) it is profit-led accumulation which will obtain.

Therefore, the meaningful subset of the domain can be divided into three regions. In the first one, comprised by low and intermediate-low levels of wage share ($\sigma < \sigma^*$), capacity utilization and accumulation are both directly related to the wage share. We refer to this region as LW in what follows. In the second region, which comprises intermediate-high levels of wage share ($\sigma^* < \sigma < \sigma^+$), even though capacity utilization is directly related to the wage share, changes in the profit share are assumed to dominate changes in capacity utilization. Within this region, therefore, referred to as IH, accumulation is inversely related to the wage share. In the third region ($\sigma > \sigma^+$), in turn, to which we refer as HW, capacity utilization and accumulation are both inversely related to the wage share.

3. *THE BEHAVIOUR OF THE MODEL IN THE LONG RUN*

In the long run we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stock of capital, K, the supply of labour, N, the price level, P, and the money wage, W. One way of following the behaviour of the system over time is by examining the dynamic behaviour of the short-run state variables σ, the wage share, and k, the ratio of capital stock to labour supply, and this is the analytical alternative pursued here. From the definition of these variables, and denoting time-rates of change by overhats, the corresponding state transition functions are:

$$\hat{\sigma} = \hat{W} - \hat{P} + \hat{a} \tag{19}$$

$$\hat{k} = \hat{K} - \hat{N} \tag{20}$$

Substitution from (11) and (12) into (10), and from the resulting expression into (19), along with substitution from (7) into (9), and from the resulting expression into (19), will yield

$$\hat{\sigma} = \mu(\rho + \lambda auk - \sigma) - \tau(\sigma - \varphi + \theta u) \tag{21}$$

where u is given by equation (14). Since we abstract from technological change, the labour-output ratio is assumed to remain unchanged throughout, which implies that the dynamics of the wage share is governed by the dynamics of the real wage.

Substituting now from (6) into (13) and the resulting expression into (20), we obtain

$$\hat{k} = (1-\sigma)u - n \tag{22}$$

where u is given by equation (14), while n is the (for now exogenous) growth rate of labour supply.

Equations (21) and (22), after using (14), constitute a planar autonomous two-dimensional system of non-linear differential equations in which the rates of change of σ and k depend on the levels of σ and k, and on the level of parameters. The matrix M of partial derivatives for this dynamic system is given by

$$M_{11} = \partial\hat{\sigma} / \partial\sigma = \mu(\lambda aku_{\sigma}{}^{*} - 1) - \tau(1 + \theta u_{\sigma}{}^{*}) \tag{23}$$

$$M_{12} = \partial\hat{\sigma} / \partial k = \mu\lambda au^{*} > 0 \tag{24}$$

$$M_{21} = \partial\hat{k} / \partial\sigma = g_{\sigma}{}^{*} \tag{25}$$

$$M_{22} = \partial \hat{k} / \partial k = 0 \qquad (26)$$

Eq. (24) shows that an increase in the capital-labour supply ratio, by raising the employment rate, will raise the wage share desired by workers, σ_w, which will raise the rate of money wage increase. Eq. (26) shows that since an increase in k does not affect either σ or u, there is no effect on the rate of accumulation, and hence no effect on the rate of growth of k.

Let us now turn to those partial derivatives whose signs are ambiguous. Eq. (23) shows that the impact of a change in the wage share on its own rate of change is mediated by the accompanying impact on capacity utilization. The reason is that the wage share desired by workers and the wage share implied by firms' desired markup depend on capacity utilization. While σ_f depends directly on the state of the goods market, σ_w depends directly on the state of the labour market. Now, given the fixed-coefficient nature of the assumed production technology, an increase in capacity utilization in the short run will necessarily be accompanied by an increase in employment. Therefore, the sign of this partial derivative will depend on the relative bargaining power of workers and capitalists. As for the sign of $\partial \hat{k} / \partial \sigma$, (25) shows that it is governed by the impact of changes in the wage share on the rate of accumulation.

We now have all the elements for a qualitative phase-diagrammatic analysis of the (local) stability properties of this dynamic system. The way we proceed is by analysing the stability of an equilibrium position in each one of the three regions into which we divided the relevant domain. Now, (17) shows that the numerator of $g = (1 - \sigma)u$ is cubic in the wage share, while its denominator is linear in the wage share. With n being exogenously given, the equation describing the $\hat{k} = 0$ isocline is cubic in the wage share as well, meaning that there may be up to three real values for the wage share in the (k - σ)-space at which a corresponding vertical $\hat{k} = 0$ isocline will be located. Given this geometry, we analyse the stability of the equilibrium position in each one of those regions were one of the $\hat{k} = 0$ isoclines to be located there.

In the LW region ($\sigma < \sigma^*$), capacity utilization and accumulation are directly related to the wage share. Hence, a higher wage share will exert an upward pressure on its own rate of change by raising capacity utilization and thus employment, which will raise the wage share desired by workers. However, this same rise in capacity utilization will also raise the markup desired by firms, which will then exert a downward pressure on the rate of change of the wage share by lowering the wage share desired by firms. Besides, the growth rate is directly related to the wage share within this region, which means that the rate of growth of *k* will rise with the wage share to make for $M_{21} > 0$. In case $M_{11} < 0$, the resulting steady-state solution will be a saddle-point. Indeed, the resulting steady-state will be a saddle-point anyway, the reason being that Det(M) < 0 no matter the sign of M_{11}.

In the IH region ($\sigma^* < \sigma < \sigma^+$), capacity utilization is still directly related to the wage share, so that the sign of $\partial\hat{\sigma}/\partial\sigma$ is as ambiguous as in the LW region. Changes in the profit share are assumed to dominate changes in capacity utilization, though, implying that the accumulation rate is inversely related to the wage share. This makes for $M_{21} < 0$, and Det(M) becomes positive. With the possibility of a saddle-point thus ruled out, whether the equilibrium solution will be stable or unstable depends on whether M_{11} is negative or positive, respectively. Therefore, stability (instability) obtains when the rate of change in prices is more (less) responsive than the rate of change in nominal wages to a change in capacity utilization generated by a change in the wage share, which makes for a negative (positive) Tr(M).

In the HW region ($\sigma > \sigma^+$), capacity utilization and accumulation are now inversely related to the wage share. As in the IH region, the accumulation rate being inversely related to the share of wages makes for Det(M) > 0, which in turn rules out the possibility of a saddle-point equilibrium. Whether the steady-state solution will be stable or unstable depends therefore on whether Tr(M) is negative or positive. Now, the sign of Tr(M) is given by the sign of M_{11}, which is again ambiguous. A higher wage share will put a downward pressure on its own rate of change by lowering capacity utilization and employment, which will lower the wage share desired by workers. However, this fall in capacity utilization will also lower firms' desired markup, which will then put a upward pressure on the rate of change of the wage share by lowering the rate of change in prices. Hence, the sign of $\partial\hat{\sigma}/\partial\sigma$ depends on the relative bargaining power of workers and capitalists, it being positive (negative) in case the fall in capacity utilization brought about by a rise in the wage share puts a stronger pressure on the rate of change of prices (nominal wages).

The relative bargaining power of capitalists and workers will thus have different stability implications in the IH ($\sigma^* < \sigma < \sigma^+$) and HW ($\sigma > \sigma^+$) regions. In the IH region, stability (instability) obtains when the rate of change in prices is more (less) responsive than the rate of change in nominal wages to a change in capacity utilization generated by a change in the wage share. In the HW region, in turn, stability (instability) obtains when the rate of change in nominal wages is more (less) responsive. The reason for this required - for stability purposes - inversion in the relative strength of the two effects is that capacity utilization and employment are directly (inversely) related to the wage share in the IH (HW) region.

More precisely, in case a rise in the wage share generates a higher (lower) capacity utilization the stability requirement is that the latter causes the rate of change in the nominal wages to rise by less (more) than the rate of change in prices. While in the LW region ($\sigma < \sigma^*$) the prevalence of wage-led growth prevents stability altogether, in the subset defined by intermediate-high (IH) and high (HW) levels of wage share the stability requirement is that the nominal wage change effect is relatively weak (strong) when wage-led (profit-led) capacity utilization prevails.

4. LONG-RUN BEHAVIOUR WITH ENDOGENOUS LABOUR SUPPLY GROWTH

The foregoing dynamic analysis was developed under the assumption that labour supply grows at an exogenous rate, which implied that desired accumulation somehow conforms to that rate.[9] We now modify the system given by (21) and (22) to endogeneize the growth rate of labour supply, while keeping intact the other building blocks of the model.

One may conceive of the flexibility of the growth of the labour supply through variations in the average age of people joining and leaving the labour force, movements into or out of the household, and migration. Thus, at least over some range, the growth of the labour force is able to adjust to the growth of the capital stock.[10] We endogeneize n along these lines by making it to depend on the dynamics of the labour market, and a reasonable assumption is to make it to depend on the employment rate:

$$n = \xi + \psi e \tag{27}$$

where ξ represents an autonomous component, ψ is a positive parameter, and e is the employment rate given by (12). The introduction of an endogenous mechanism of labour supply growth allows the labour supply to adjust, at least over some range, to meet labour demand in the long run. Hence, the long-run equilibrium is determined mostly by capital accumulation: to some extent, the natural rate of growth adapts to the warranted rate.[11]

9 Which does not mean, however, that a model of this kind is supply-constrained in the usual sense. (DUTT, 1994) First, the economy still has unemployed labour at long-run equilibrium. The rate of employment has stabilized to stabilize the wage share, which is required for long-run equilibrium. Second, unlike models with exogenously-given growth, this model has examined the dynamics of the economy out of long-run equilibrium in which it is driven by demand as well as supply forces. Since the economy will arrive, if at all, at long-run equilibrium only in the limit, and since exogenous parametric shifts may be shifting this equilibrium frequently, the economy is not usually supply-constrained in the sense that it always grows at the rate of labour supply growth.

10 According to SAWYER (1989), the supply of labour to the capitalist economies (and within capitalist economies, supply to the capitalist sectors) can, at least over some range, be readily expanded whenever it is necessary. Within a country, the capitalist economy may cover only a part of the economy, so that the capitalist sector can pull workers from the non-capitalist sector when demand for labour is relatively high and push workers back when demand is low. Other mechanisms include migration of labour from one country to another and changes in the age of entry into and departure from the labour force. Therefore, extra supply of labour can be obtained when demand is strong by pulling people into the labour force. Conversely, when the overall demand for labor is low, unemployment can to some degree be hidden by the re-absorption of workers back into home.

11 LEÓN-LEDESMA & THIRLWALL (2000, 2002) estimate the sensitivity of the natural rate of growth to the actual rate of growth for 15 OECD countries over the period 1961 to 1995 and conclude that both components of the former (labour force growth and labour productivity growth) are endogenous to output growth. In their view, there are a variety of ways, well documented, by which the growth of labour inputs increases when output growth is buoyant: hours worked increase; participation rates increase, particularly among females; reallocation of labour from low to high productivity sectors take place, which is a very important factor in the early stages of industrialization; and immigration may also occur.

A possible rationale for this specification is that the tighter the labour market, the higher the prospects of finding - and keeping - a position for for those outside the labour force, which will then animate them to (re)join the labour force at the prevailing nominal wage. Indeed, this specification is more inclusive than the preceding one in the sense that it incorporates a feedback effect from the dynamics of the goods and labour markets to the dynamics of the labour force growth. Now a higher rate of employment will not only induce those already employed to conduct nominal wage bargain in a way that ensures a (potentially) higher wage share, but will also stimulate those outside the labour force to (re)join it.[12]

Having been modified in this manner, our dynamic system is now given by

$$\hat{\sigma} = \mu(\rho + \lambda auk - \sigma) - \tau(\sigma - \varphi + \theta u) \tag{21}$$

$$\hat{k} = (1-\sigma)u - \xi - \psi auk \tag{28}$$

where u is given by (14). Again, we have an autonomous two-dimensional system in which the rates of change of the state variables σ and k depend on the levels of σ and k, and on the level of parameters. The matrix M^{+} of partial derivatives for this system is:

$$M_{11}{}^{+} = \partial\hat{\sigma} / \partial\sigma = \mu(\lambda aku_{\sigma}{}^{*} - 1) - \tau(1 + \theta u_{\sigma}{}^{*}) \tag{29}$$

12 VELUPILLAI (1993) develops an interesting model along classical lines by re-specifying JOHANSEN's (1967) formulation of the classical model. In this re-specification, the growth rate of labour supply is positively related to the rate of growth of the money wage and the share of wages in income. With the growth rate of money wages being positively related to the employment rate, the growth rate of labour supply becomes positively related to the employment rate and the wage share. For Velupillai, these assumptions are actually mathematically equivalent to assuming a given bargained money wage determining a lower limit and the growth of the market rate being, then, a function of the employment rate. In the model of this paper, which introduces explicitly real wage dynamics through a conflict theory of inflation, the growth rate of nominal wages is positively related to the employment rate through an endogenously determined desired wage share. Capacity utilization and the rate of accumulation being variable, the rate of employment depends non-linearly on the wage share. Hence, it seems reasonable to make the growth rate of labour supply to depend on the employment rate only.

$$M_{12}{}^{+} = \partial\hat{\sigma} / \partial k = \mu\lambda a u^{*} > 0 \tag{30}$$

$$M_{21}{}^{+} = \partial\hat{k} / \partial\sigma = g_{\sigma}{}^{*} - \psi a k u_{\sigma}{}^{*} \tag{31}$$

$$M_{22}{}^{+} = \partial\hat{k} / \partial k = -\psi a u^{*} < 0 \tag{32}$$

As the state transition function for the wage share given by (21) has not changed, the sign of $\partial\hat{\sigma} / \partial\sigma$ is as ambiguous as in the preceding system, while the sign of $\partial\hat{\sigma} / \partial k$ is likewise positive. Since the rate of change in $\hat{k}$ was now made to depend on the level of *k* as well, the other partial derivatives will change with respect to the foregoing analysis. Since we have endogeneized the rate of growth of labour supply, the sign of $\partial\hat{k} / \partial\sigma$ becomes more ambiguous. A change in the wage share, by changing capacity utilization, will now affect both the rate of growth of the capital stock and the rate of growth of labour supply, and we analyse this ambiguity for each region of the domain in detail below. As for the sign of $\partial\hat{k} / \partial k$, it is unambiguously negative. For any change in capacity utilization, by changing the rate of employment, will cause a change in the same direction in the growth rate of labour supply and, therefore, will cause a change in the opposite direction in the rate of growth of the ratio of capital stock to labour supply.

We now have all the elements for a qualitative phase-diagrammatic analysis of the (local) stability properties of this modified system. As before, we proceed by analysing the stability of the equilibrium in each one of the three regions in which we divided the relevant domain. The equation describing the $\hat{k} = 0$ isocline became dependent on the level of k, though, thus implying that there is a single non-vertical $\hat{k} = 0$ isocline in the (k - σ)-space. This makes for the possibility of multiple equilibria, and we turn to this possibility in the next section.

In the preceding specification of the model, the steady-state solution for the LW region ($\sigma < \sigma^{*}$) would necessarily be a saddle-point, the reason being that Det(M) < 0 no matter the sign of $\partial\hat{\sigma} / \partial\sigma$. In the present specification with an endogenous labour supply growth, that is not necessarily the case anymore. Recall that in this region $u_\sigma^* > 0$ and $g_\sigma^* > 0$, thus implying that the sign of M_{21}^+ will depend on the relative strength of these two effects. Now, recall that for a higher wage share to raise the growth rate, it has to generate a rise in capacity utilization which more than compensates the accompanying fall in the share of profits in income, which means that $u_\sigma^* > g_\sigma^*$. Therefore, we will have $M_{21}^+ < 0$ unless the response of labour supply growth to a change in capacity utilization is low enough to ensure that the growth effect more than compensates the capacity utilization effect on $\hat{k}$, despite $u_\sigma^* > g_\sigma^*$. Given the strength of the endogenous labour supply effect, chances for a negative sign for M_{21}^+ are the higher, the greater the extent to which u_σ^* is stronger than g_σ^*.

Let us then analyse the situation in which $M_{21}^+ < 0$ and $M_{11}^+ < 0$, the latter meaning that the rate of price change is more responsive than the rate of change in nominal wages to changes in capacity utilization. In this case, the economy has a stable long-run equilibrium given that Det(M^+) is positive and Tr(M^+) is negative. Therefore, while in the preceding model specification the equilibrium solution was not stable in the subset of the domain in which wage-led accumulation ($g_\sigma^* > 0$) would obtain, the introduction of endogenous labour supply growth makes for a stable system - provided the rate of change in prices is more responsive than the rate of change in nominal wages to changes in capacity utilization, on the one hand, and the response of the growth rate of labour supply to a change in capacity utilization is not too low and/or the extent to which u_σ^* is greater than g_σ^* is not too low, on the other hand.

However, the situation may change in case $M_{11}^+ > 0$, meaning that the rate of price change is less responsive than the rate of change in nominal wages to changes in capacity utilization, and $M_{21}^+ < 0$. The reason is that both the sign of Det(M^+) and the sign of Tr(M^+) become unclear now, which will require a closer examination of their formal expressions given by

$$Det(M^+) = au^*[(\mu + \tau + \tau\theta u_\sigma^*)\psi - \mu\lambda g_\sigma^*] \tag{33}$$

$$Tr(M^+) = \mu(\lambda a k u_\sigma^* - 1) - \tau(1 + \theta u_\sigma^*) - \psi a u^* \tag{34}$$

Now, recall that for a higher wage share to raise the accumulation rate, it has to generate a rise in capacity utilization which more than offsets the accompanying fall in the profit share, which means that $u_\sigma^* > g_\sigma^*$. Since we are analysing the situation in which $M_{11}^+ > 0$, it follows that $Det(M^+) > 0$ and $Tr(M^+) < 0$ unless the response of labour supply growth to a change in the rate of employment is too low and/or the response of workers' desired wage share to a change in capacity utilization is too strong. Otherwise, either $Det(M^+)$ will be negative and saddle-point instability will obtain no matter the sign of $Tr(M^+)$, or $Det(M^+)$ will be positive but the sign of $Tr(M^+)$ will be positive as well and an unstable solution will follow. Hence, a stable equilibrium solution becomes possible even in case the rate of change in nominal wages happens to be more responsive than the rate of change in prices to changes in capacity utilization.

In case $M_{11}^+ < 0$ and $M_{12}^+ > 0$, eq. (34) shows that the sign of $Tr(M^+)$ will be unambiguously negative, which means that whether the equilibrium solution will be stable or saddle-point unstable depends on whether $Det(M^+)$ will be positive or negative. Now, $M_{11}^+ < 0$ suggests that the response of the rate of nominal wage change to a change in capacity utilization is weak, while $M_{21}^+ > 0$ suggests that the response of the rate of growth of labour supply to a change in capacity utilization is weak as well. Given that $u_\sigma^* > g_\sigma^*$, (33) shows that $Det(M^+) > 0$ unless ψ is much lower than λ. In the event ψ is sufficiently lower than λ to make for a negative $Det(M^+)$, the equilibrium will be saddle-point unstable.

Finally, equilibrium will be saddle-point unstable in the event M_{11}^+ and M_{21}^+ are positive, the reason being that now $Det(M^+)$ will be negative. Hence, a weak endogenous labour supply growth effect coupled with a strong nominal wage growth effect makes for a saddle-point unstable system.

In the IH region ($\sigma^* < \sigma < \sigma^+$), capacity utilization is directly related to the wage share, so that the sign of M_{11}^+ is ambiguous as before. The sign of M_{21}^+ is negative, the reason being that g_σ^* is negative in this subset of the relevant domain. In case M_{11}^{+} is negative, stability will automatically obtain. In case M_{11}^{+} is positive, meaning that the nominal wage change effect is stronger than the price change effect, (33) shows that $Det(M^{+}) > 0$, and whether stability or instability will obtain depends on the sign of $Tr(M^{+})$, which in turn depends on the relative strength of the endogenous labour supply effect with respect to the nominal wage effect. In case the extent to which the nominal wage change effect is greater than the price change effect is larger (smaller) than the response of the rate of change in labour supply to a change in the rate of employment, instability (stability) will obtain. In this subset of the relevant domain, therefore, the introduction of an endogenous mechanism of labour supply growth clearly makes for a more resistant system, in that it takes a nominal wage growth effect stronger than before to cause instability.

In the HW region ($\sigma > \sigma^{+}$), both capacity utilization and accumulation are inversely related to the wage share. The sign of M_{21}^{+} is ambiguous, for while $g_{\sigma}^{*} < 0$ is making for a negative $\partial\hat{k} / \partial\sigma$, $u_{\sigma}^{*} < 0$ is rather making for a positive one. However, $g_{\sigma}^{*} > u_{\sigma}^{*}$ because, for instance, a falling capacity utilization is actually adding to a falling profit share to make for a falling growth rate. Hence, chances for a negative M_{21}^{+} are the higher, the weaker the response of the labour supply growth to a change in capacity utilization and/or the more g_{σ}^{*} is greater than u_{σ}^{*}. In case M_{21}^{+} is negative and $M_{11}^{+} < 0$, which now means that the rate of change in nominal wages is more responsive than the rate of change in prices to a change in capacity utilization, Det(M^{+}) is positive and Tr(M^{+}) is negative, which in turn makes for a stable equilibrium solution within this subset of the domain. In case M_{21}^{+} is negative but the price change effect is greater than the nominal wage change effect, which makes for $M_{11}^{+} > 0$, both the sign of Det(M^{+}) and the sign of Tr(M^{+}) become unclear. In this case,

(33) and (34) show that chances for $Det(M^{+}) > 0$ are the higher, the greater the extent to which $g_{\sigma}^{*} > u_{\sigma}^{*}$, while chances for $Det(M^{+}) > 0$ and for $Tr(M^{+}) < 0$ together are the higher, the greater the extent to which $g_{\sigma}^{*} > u_{\sigma}^{*}$, the smaller the extent to which the price change effect is greater than the nominal wage change effect and the stronger the endogenous labour supply growth effect. In the event the relative strength of these latter two effects - and of u_{σ}^{*} and g_{σ}^{*} - is such that $Det(M^{+}) > 0$ but $Tr(M^{+}) > 0$, an unstable equilibrium will obtain. For instance, instability would obtain in the event the price change effect is strong enough to dominate the endogenous labour supply effect in the sign of Tr(M^{+}), even though it is not strong enough to make for a negative Det(M^{+}). In case the relative strength of all these effects is such that $Det(M^{+}) < 0$, then a saddle-point unstable equilibrium will obtain. Basically, chances for a negative Det(M^{+}) are the higher, the smaller the extent to which $g_{\sigma}^{*} > u_{\sigma}^{*}$, the greater the extent to which the price change effect is greater than the nominal wage change effect and the stronger the endogenous labour supply effect.

As mentioned above, the sign of M_{21}^{+} will be negative unless the response of the labour supply growth to a change in capacity utilization (and employment) is very high and/or the extent to which $g_{\sigma}^{*} > u_{\sigma}^{*}$ is small enough. Combined with $M_{11}^{+} < 0$, which in this subset of the domain means that the nominal wage change effect is stronger than the price change effect, a positive sign for M_{21}^{+} will make for an ambiguous sign for $Det(M^{+})$. The sign of $Tr(M^{+})$ is negative, though, so that the equilibrium solution will be either stable or saddle-point unstable. Eq. (33) shows that a positive sign for $Det(M^{+})$ will obtain unless the extent to which $g_{\sigma}^{*} > u_{\sigma}^{*}$ is very small and/or the endogenous labour supply effect is significantly stronger than the nominal wage effect. Finally, equilibrium will be unambiguously saddle-point unstable in the event $M_{21}^{+} > 0$, while the price change effect is stronger than the nominal wage effect, which makes for $M_{11}^{+} > 0$.

5. *MULTIPLE EQUILIBRIA ANALYSIS*

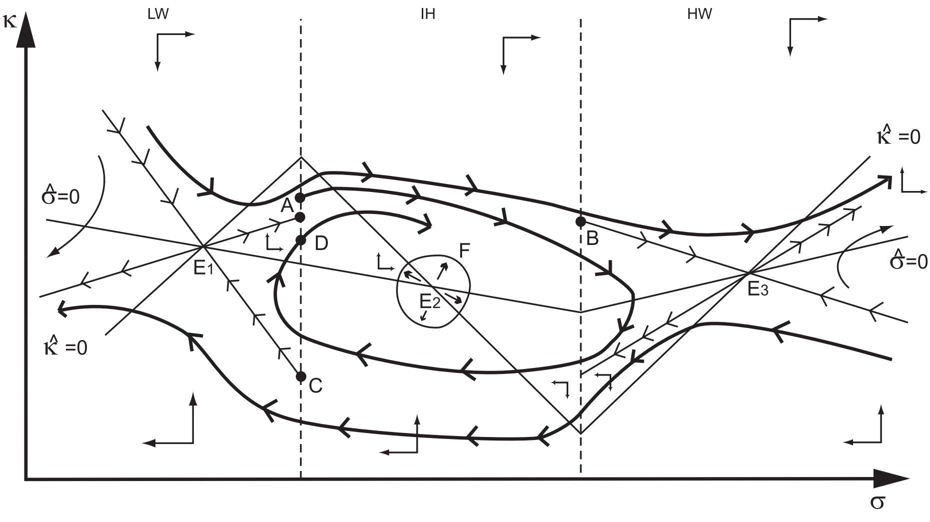
The non-linearity embodied in the desired investment function makes for the possibility of multiple equilibria within the relevant domain, this being the case with either exogenous or endogenous labour supply growth. As shown above, with an exogenously given growth rate of labour supply there may be up to three real values for the share of wages at which a corresponding vertical $\hat{k} = 0$ isocline would be located in the $(k - \sigma)$-space, so that it is possible that a configuration with three equilibria obtains within the relevant domain.

Since a broader set of possible multiple equilibria configurations was open by the introduction of an endogenous mechanism of labour supply growth, it is on this case that we will focus more closely in this section. Amongst the possible configurations leading to multiple equilibria, one worthy of a more detailed phase-diagrammatic analysis contains a saddle-point in the LW region, a stable or unstable solution in the IH region and another saddle-point in the HW region. Now, while the sign of $M_{11}^{+} = \partial\hat{\sigma} / \partial\sigma$ depends on the relative bargaining power of capitalists and workers, the sign of $M_{21}^{+} = \partial\hat{k} / \partial\sigma$ depends on the relative response of the accumulation rate and of the growth rate of labour supply to a change in the share of wages. Let us hypothesize that the parameters that govern the relative bargaining power of capitalists and workers can be such that, in response to a change in the wage share, the nominal wage change effect is greater than the price change effect throughout the relevant domain, which implies that $M_{11}^{+} = \partial\hat{\sigma} / \partial\sigma$ is positive in the LW and IH regions and negative in the HW region.

Regarding the sign of $M_{21}^{+} = \partial\hat{k} / \partial\sigma$, let us hypothesize the following. (31) shows that M_{21}^{+} will be positive in the LW region provided u_{σ}^{*} is not much higher than g_{σ}^{*} and/or the endogenous labour supply effect is not too strong. In other words, M_{21}^{+} will be positive in the event the rate of accumulation is more responsive than the rate of growth of labour supply to a change in the wage share. In the IH region, in turn, it was seen above that M_{21}^{+} will be unambiguously negative no matter the relative strength of the several effects in action. Finally, (31) shows that M_{21}^{+} will be positive in the HW region provided g_{σ}^{*} is not much higher than u_{σ}^{*} and/or the endogenous labour supply effect is not too weak. Recalling that the relative strength of all these effects depends on the levels of σ and k as well as on parameters of the model, let us hypothesize that those levels and parameters can be such that M_{21}^{+} is positive in the LW and HW regions.

Hence, we have a situation in which $M_{11}^{+} = \partial\hat{\sigma} / \partial\sigma$ is positive in the LW and IH regions and negative in the HW region, whereas $M_{21}^{+} = \partial\hat{k} / \partial\sigma$ is positive in the LW and HW regions and negative in the IH region. Recalling that $M_{12}^{+} > 0$ and $M_{22}^{+} < 0$ throughout the domain, a situation like the one being hypothesized here is pictured in Fig. 1. In the LW region, saddle-point instability will obtain, while whether stability or instability will obtain in the IH region depends on the relative strength of $M_{11}^{+} > 0$ and $M_{22}^{+} < 0$, which will give the sign of $Tr(M^{+})$, and we analyse these two possible cases in what follows. In the HW region, (33) shows that chances for a negative value for $Det(M^{+})$, which will make for saddle-point instability, will be the higher, the stronger the endogenous labour supply growth effect, the smaller the extent to which g_{σ}^{*} is greater than u_{σ}^{*}, and the smaller the extent to which the nominal wage change effect is greater than the price change one. Recalling that the relative strength of all these effects depends on the levels of σ and k as well as on parameters of the model, let us hypothesize that those levels and parameters can be such that $Det(M^{+})$ is negative in this subset of the relevant domain.

FIGURE 1 - MULTIPLE EQUILIBRIA, STABILITY ZONE AND CYCLICAL BEHAVIOUR



Therefore, an interesting configuration with three equilibria would obtain within the relevant domain, and let us call E_1, E_2 and E_3 the equilibrium solutions located in the LW, IH and HW regions, respectively. Having analysed these three equilibria in isolation in the previous section, we now perform a qualitative phase-diagrammatic analysis of the whole domain. Indeed, we are in position to reveal an interesting potentially cyclical feature of equilibrium points such as E_2. Suppose we begin a trajectory at point A in Fig. 1. The direction of motion of the system indicates that it must flow rightward up until the $\hat{k} = 0$ isocline is reached, after which the system will flow rightward down. Recall that in the IH region an increase in the wage share will raise capacity utilization but lower the rate of accumulation. Before the $\hat{k} = 0$ isocline is crossed, though, the net impact of these effects is to raise the level of k, with the reason being the following. While (31) shows that the rise in capacity utilization and the fall in the rate of accumulation are putting a downward pressure on the level of k, (32) shows that this fall in the level of k will raise the rate of growth of k more than proportionately to make for a resulting rise in the level of k. Once the $\hat{k} = 0$ isocline is reached, though, further increases in the wage share will lead to a fall in the level of k.

Now, recall that the fixed-proportion nature of the production technology implies that capacity utilization and the rate of employment move in the same direction, with the strength of this connection being given by ak. In other words, changes in k lead to changes in the relative impact of higher capacity utilization on the share of wages desired by workers and on the markup desired by firms. Besides, what makes a rise in the wage share even more self-undermining is that by raising capacity utilization and thus employment, it raises the rate of growth of labour supply. Since we have hypothesized above that in response to a change in the wage share the nominal wage change effect is stronger than the price change effect, though, the system will keep flowing rightward down until it reaches either the HW region or the $\hat{\sigma} = 0$ isocline in the same IH region.

Suppose the system reaches the HW region before reaching the $\hat{\sigma} = 0$ isocline. In case it reaches that region through anywhere above point B, the system will keep flowing rightward down until it reaches the $\hat{k} = 0$ isocline, after which it will flow rightward up and further away from E_3 - and actually from anyone of the other equilibria. Indeed, it is only by a happy fluke that the system will enter the HW region through point B, which would take it to E_3. Suppose the system reaches that region through some point below B. Once inside the HW region, it will keep flowing rightward down until the $\hat{\sigma} = 0$ isocline is reached, where the motion of the system will then undergo an important qualitative change. Recall that in this subset of the domain capacity utilization and accumulation are both inversely related to the wage share. Before the $\hat{\sigma}=0$ isocline is reached, an increase in the wage share, by lowering the rate of accumulation by more than it lowers the growth rate of labour supply, will lower the level of k. However, it is only when the system reaches the $\hat{\sigma}=0$ isocline that σ is high enough - and k is low enough - for a further increase in the wage share to have a negative impact on its rate of growth that more than offsets that increase. With the wage share now falling, capacity utilization and accumulation both start rising again, even though k keeps falling, since the rise in capacity utilization raises the growth rate of labour supply by more than the rise in the accumulation rate.

The system will flow leftward down until it reaches back the IH region, after which it will keep such motion for a while. Even though capacity utilization is falling and profit-led accumulation prevails, the levels of σ and k have not fallen enough yet to ensure the reversal of the leftward down motion of the system. Once it reaches the $\hat{k}=0$ isocline, though, a decrease in the wage share, despite leading to a further decrease in it, will now raise the level of k, so that the system will start moving leftward up. A happy fluke may take the system to point C, which will then lead it to converge to E_1. In the event the system reaches the LW region through a point below C, it will flow leftward up and then, having crossed the $\hat{k}=0$ isocline, will flow leftward down, with falling levels of both σ and k. Given the saddle-point nature of E_1, the level of k could not rise to the extent that was necessary to allow the wage share to start increasing again.

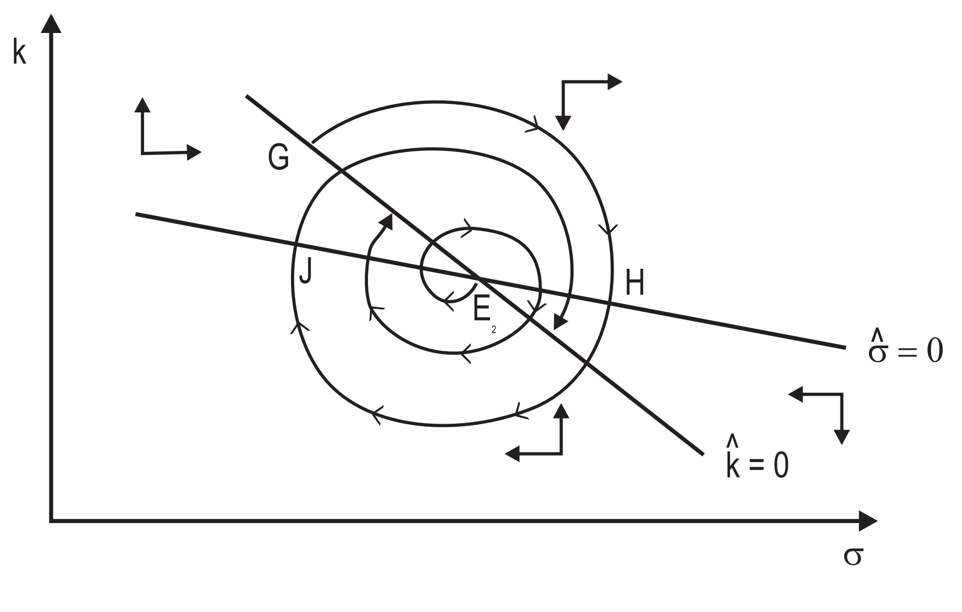
Let us suppose the system enters the LW region through a point above C, and recall that both capacity utilization and accumulation are wage-led in that region. Since the former is dominating the latter in the growth rate of $\hat{k}$, the level of k can keep rising and thus reach the $\hat{\sigma}=0$ isocline, after which the wage share will start rising again. Besides, since the capacity utilization effect is still being dominated by the accumulation effect in $\hat{k}$, the level of k keeps rising. Once the system reaches back the

IH region - through, say, point D - the cyclical motion just described will re-start. However, this inner part of the cyclical trajectory will not cross the previous one, since trajectories of differential equations with continuous partial derivatives must be unique. (ARROWSMITH & PLACE, 1992) Also, this inner part may or may not pass through the HW and LW regions again, it being possible that the two changes of sign of $\hat{\sigma}$ occur within in the IH region itself. Now, recall that whether E_2 is stable or unstable depends on the relative strength of the positive $\partial\hat{\sigma}/\partial\sigma$ with respect the negative $\partial\hat{k}/\partial k$. More precisely, E_2 will be locally stable (unstable) in case the response of the rate of growth of labour supply to a change in the level of k is stronger (weaker), in absolute value, than the extent to which the response of the rate of nominal wage change is stronger than the rate of price change to a change in the level of σ.

In the event E_2 is locally stable, there is a region around it in which all trajectories tend to E_2. Since the system will (eventually) end up reaching that region along the trajectory started at A, it will converge to E_2 . In case E_2 is unstable, which is the situation pictured in Fig. 1, there is a neighborhood of E_2, say F, in which all trajectories of the system will move away from E_2. Since the system will (eventually) end up reaching that neighborhood along the trajectory initiated at point A, it will not reach E_2. Indeed, there may eventually be a closed, bounded area encircling the neighborhood F and from which no trajectory will exit. In case this area contains no equilibrium points, the Poincaré-Bendixson theorem would ensure that it must then contain at least one stable limit cycle: any point in the area not on a limit cycle would be attracted to such a cycle. (ARROWSMITH & PLACE, 1992) Whether or not some limit cycle will emerge, though, the system will move cyclically in the IH region, which shows its propensity to experience endogenous, self-sustaining fluctuations in the capital to labour supply ratio and distributive shares, with capacity utilization, accumulation and employment rate fluctuating as well.

Suppose, for instance, that the economy starts from point G in Fig. 2. A rise in the wage share will raise capacity utilization and lower the accumulation rate, which will then put an unambiguously downward pressure on the level of k. Since the response of the rate of change in nominal wages is stronger than the response of the rate of price change to a change in capacity utilization, this rise in the wage share will engender a further increase of it. Now, this fall in k will put a downward pressure on the wage share by lowering the employment rate for a given degree of capacity utilization. But this fall in k will also put an upward pressure on itself because the fall in the employment rate will lower the growth of labour supply for a given rate of accumulation. Nonetheless, the level of k is still high enough and the level of σ still low enough to ensure that a fall in the former and a rise in the latter follows. As the system approaches point H, though, the extent to which the nominal wage effect is stronger than the price change effect in $\hat{\sigma}$ is falling.

FIGURE 2 - LONG-RUN DYNAMICS AND CYCLICAL BEHAVIOUR



Once point H is reached, the wage share (capital to labour supply ratio) will have risen (fallen) by enough to make for a stationary wage share.

However, k is still falling, the reason being that it has not fallen enough to engender a reversal of such downward trend. With the wage (profit) share falling (rising) now, a falling capacity utilization and a rising rate of accumulation are putting an upward pressure on k, while a rise in k, by raising the employment rate for a given capacity utilization, will put a downward pressure on k by raising the growth of labour supply for a given accumulation rate. Once the system reaches point I, though, k and σ will have fallen by enough for k to be stationary. But since the wage share is falling, k will soon start rising, the reason being that at those levels of σ and k the accumulation effect will soon become stronger than the labour supply effect in $\hat{k}$. As the system approaches point J, though, the extent to which the nominal wage effect is stronger than the price change effect in $\hat{\sigma}$ is rising. Once point J is reached, the values of the two state variables will be such that the downward trend in the wage share will cease. At this point the cyclical motion just described will re-start.

Hence, this model shares with the classic contribution by Goodwin (1967) a cyclical growth dynamics governed by the interaction between accumulation of capital, distribution and employment.[13] Unlike the Goodwin model, though, this one allows effective demand to play an active role through a variable degree of capacity utilization, incorporates price and nominal wage dynamics through a conflict theory of inflation, and introduces a feedback from the dynamics of the labour and goods market to the growth of labour supply. If left undisturbed, the Goodwin model will

13 In the Goodwin model, the share of wages is determined by the reserve army of labour, or, more precisely, by the employment rate. The pace of capital accumulation determines the demand for labour. If the rate of accumulation is rising sufficiently, so does the employment rate. Beyond a certain value of the employment rate (i.e. in the neighborhood of full employment) the vigorous accumulation of capital leads to a rising real wage and a rising wage share. This process goes on until the rise in the wage share is sufficient to reduce the rate of profit to a point where the rate of accumulation slows down and unemployment begin to rise. The replenishment of the reserve army of labour yields a falling wage share, therefore a rising rate of profit and eventually an upturn in the rate of accumulation.

produce conservative cyclical fluctuations in distributive shares and in the rate of employment. However, the trajectories will no longer be closed orbits if direct feedbacks from the wage share to its rate of growth - or from the level of the employment rate to its rate of growth - are introduced.

The model of this paper introduces both feedbacks via variable capacity utilization, conflict inflation and endogenous labour supply growth. Indeed, the first two features are shared with Dutt (1992, 1994), from which this model has drawn a lot of inspiration. Unlike the latter, though, this model does not rely on full capacity being reached for profit-led capital accumulation and multiple equilibria to obtain. Given the non-linear nature of the investment function used here, the system may experience self-sustaining fluctuations in the state variables - eventually alternating phases of wage-led accumulation with phases of profit-led accumulation - below full capacity utilization. Indeed, the non-linear nature of the model allows it to specify precise ranges of distributive shares within which wage-led and profit-led capacity utilization and accumulation obtain.

SUMMARY

This paper developed a post-keynesian model of capital accumulation, distribution and conflict inflation in which desired investment is made to be non-linear in distributive shares, an specification that seems to conform with the empirical evidence on the rise and fall of the Golden Age in most advanced economies. Whether the economy will follow a wage-led accumulation regime or a profit-led one depends on the prevailing distribution, with a similar dependence applying - along with the relative bargaining power of workers and capitalists and whether the growth rate of labour supply is exogenous or endogenous - to its dynamic stability properties. While wage-led capacity utilization and growth will obtain only for sufficiently low levels of wage share, profit-led capacity utilization and growth will obtain only for sufficiently low levels of profit share. Hence, an innovative feature of this model is that it does not rely on full

capacity utilization being reached for a change in the capital accumulation and growth regimes to take place. Given such non-linearity, the paper closed with a qualitative, phase-diagrammatic analysis of a possible configuration for the system leading to multiple equilibria and endogenous, self-sustaining fluctuations in its relevant variables.

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