Liquidity Constraints and the Behavior of Aggregate Consumption over the Brazilian Business Cycle

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RESUMO

Uma característica marcante no ciclo econômico brasileiro é a alta volatilidade do consumo. O desvio padrão do ciclo do consumo de bens não duráveis, no período entre 1970 e 1998, é igual a 5,26%, quase igual ao do produto (5,57%). Um dos motivos pelos quais os modelos de ciclos reais de negócios não conseguem reproduzir este fato pode ser a existência de restrições ao crédito encontrada pelos consumidores. Este trabalho apresenta uma versão do modelo básico de ciclos reais onde uma fração dos agentes é restrita a consumir toda sua renda a cada período. Simulações numéricas mostram que o modelo com restrição ao crédito é capaz de reproduzir a alta volatilidade da série de consumo, porém o modelo subestima a correlação entre investimento e produto.

PALAVRAS-CHAVE

consumo de bens não-duráveis, modelo de ciclos reais de negócio, volatilidade

ABSTRACT

The Brazilian business cycle presents a key feature consisting of a high volatility of the consumption series. The standard deviation of the non-durable consumption series reaches 5.26%, almost as high as the volatility of the GNP (5.57%) over the period 1970-1998. The failure of the standard real business cycle model to capture this fact could be related to the high credit restriction most consumer faces in the country. The present study aims to present an extended recursive general equilibrium model to better mimic the above empirical evidence, introducing heterogeneous agents in the model economy characterized by a set of agents who does not behave according to the permanent income hypothesis due to the credit restriction they face every period. The numerical analysis of the proposed model economy reproduces the high volatility present in the Brazilian consumption series. However, the correlation between output and investment series is underestimated due to the presence of those agents who cannot smooth out consumption over the business cycle.

KEY WORDS

non-durable consumption, real business cycle model, volatility

JEL Classification

E32, E13

INTRODUCTION

One of the key features present in the Brazilian business cycle is the high volatility of the aggregate consumption series.[1] Ellery Jr., Gomes and Sachsida (2002) show that, for Brazil, the standard deviation of the aggregate non-durable consumption series (5.16%) is as high as the one corresponding to the aggregate output series (5.57%), the correlation coefficient between the series being 0.8087.

The standard model used in the real business literature is not able to capture the above stylized fact due mainly to the theoretical assumption according to which agents optimally are able to smooth out consumption over the business cycle produced by the model economy. In other words, typically in those models, agents can behave according to the permanent income hypothesis because markets are complete; therefore, they are able to finance their "smooth" sequence of consumption despite their volatile income, such that the expected present value of their lifetime utility stream can be maximized.

Issler and Rocha (2000), using the partial equilibrium framework suggested by Campbell and Mankiw (1989) and the Brazilian data set, estimated the share of liquidity-constrained consumers to be as high as 75% of the consumers in the country. Reis et all (1999) reached also o similar magnitude. These studies strongly suggest, first, the need to introduce a set of credit-constrained agents into the model economy in order to capture this restriction faced by most agents and, second, that this credit restriction could be the cause of the high volatility of the aggregate consumption series present in the Brazilian empirical evidence.[2]

1 MENDOZA (1995) and CARMICHAEL, KÉITA & SAMSON (1999) show that this fact is common among developing countries but durable consumption is included in the aggregate consumption series used.

2 The relevance of liquidity constraints is clearly stated and analyzed by CHARMICAEL, KÉITA & SAMSON (1999), using an overlapping generations framework for small open developing countries to mimic the small positive correlation between the trade balance and the terms of trade as well as a small negative correlation between the former and the aggregate output.

Based on the above arguments, the present study aims to construct a dynamic general equilibrium model, which could reproduce the key characteristic of the Brazilian business cycle regarding the high volatility of the aggregate consumption series. To this end, a basic model of infinitely lived agents is extended to introduce heterogeneous agents, for a large group of them are restricted to consume all their income at every period.[3] Once the model is calibrated with parameter values consistent with the Brazilian aggregate data, the simulation results show that the model can better mimic the Brazilian key stylized fact. Indeed, the model generates an aggregate consumption series as volatile as the aggregate output series.

In some sense the structure of the herein considered model is a simplified version of the one presented in Carmichael, Kéita and Samson (1999).[4] These authors suggest a model where some individuals are restricted to consume their entire income at each period in the same fashion as the restricted agents of the model in this paper. Nevertheless, their artificial economy consisting of a small open economy model with an overlapping generation structure with three different sectors operating in the productive sector at every period, aims to study the volatility of the balance of payments, which is a different objective from the one proposed in this paper.

For the particular question at hand, namely the high aggregate consumption volatility present in the Brazilian data, the herein implemented model economy represents a closed economy with infinitely lived agents. There is only one good that may be produced by firms that can operate just one of the two available technologies. This characterization introduces the needed sufficient friction into the model economy to analyze the aggregate consumption volatility present in the Brazilian data.

3 Future work should be made in order to create a version of the model where the agents choose to consume theirs income at each period.

4 We thank this comment to an anonymous referee. However, while it is true that the model is a simplification of the model in CARMICHAEL, KÉTIA & SAMSON (1999) it is important to remark that it was constructed and proposed as a dynamic general equilibrium version of the model in CAMPBELL & MANKIW (1989).

The paper is organized as follows. The next section will provide some statistical properties of the Brazilian aggregate consumption series. Particularly, the volatility of this series *vis-à-vis* the volatility of the aggregate output series will be emphasized. Section 2 will present the extended stochastic general equilibrium model as well as the definition of the recursive equilibrium, which will be numerically solved for. Section 3 will introduce the calibration and the results of the simulations. Finally, the last section will discuss the main conclusions and some suggestions for future research.

1. *STATISTICAL PROPERTIES OF THE BRAZILIAN AGGREGATE CONSUMPTION SERIES*

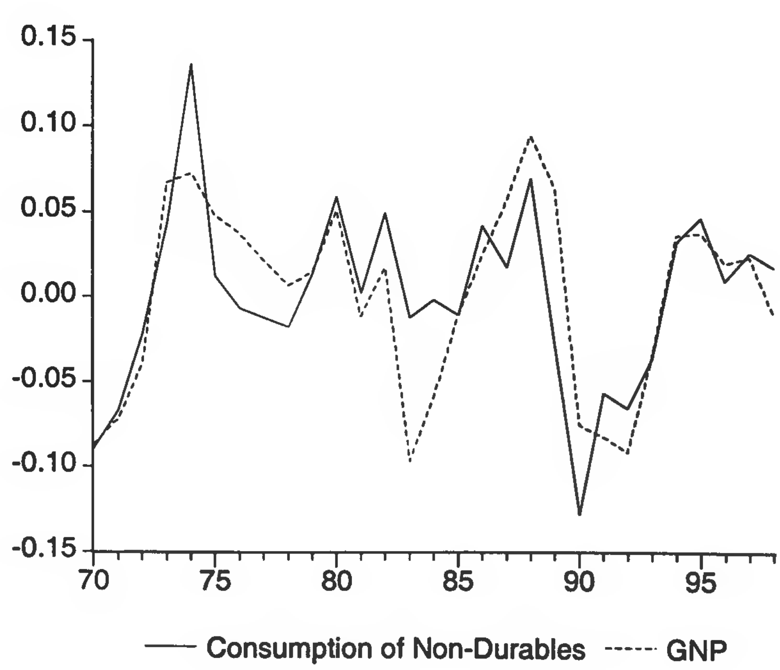
As for many developing countries, Brazil's aggregate consumption series presents a high volatility as well as a high correlation with aggregate output. Camichael, Kéita and Samson (1999) report a business statistics table for a set of developing countries and using data from the International Financial Statistics (IMF) estimated a standard deviation of 13.16% for the Brazilian consumption series and a correlation coefficient with real GDP of 0.866 for the period 1968-1993. However, the data on consumption includes durable goods as well as the variation on inventory stock since 1986, which is not theoretically consistent with the model economy employed in the business cycle literature.

In order to overcome with the above inconsistency in the data for working with business cycle models, Ellery Jr, Gomes and Sachsida (2002) elaborated a series using the Brazilian National Account Tables (IBGE) covering the period 1970-1998. Even though this data source presents a methodological problem for, as mentioned above, durable consumption and sinc 1986 changes in inventory stock are included, they skillfully end up elaborating a series of aggregate consumption series consistent with its theoretical definition used in the real business cycle models. The authors exclude those components from the original series using the Input-Output matrices (IBGE) of 1970, 1975, 1980, 1985 and 1990 to 1998. According to their study, the standard deviation corresponding to the non-durable consumption series is 5.25% whereas the durable consumption series presents a higher volatility of 11.43% and, the cross-correlation of the former with output of 0.8087.

The above two references clearly suggest that Camichael, Kétia and Samson (1999) over-estimate the volatility of the Brazilian aggregate consumption relevant for the business cycle model as well as the correlation of consumption to the aggregate output. Therefore, for the purpose of the present article, the statistical properties of the Brazilian aggregate non-durable consumption presented by Ellery Jr., Gomes and Sachsida (2000) will be adopted.

Figure 1 below graphically introduces the series of non-durable consumption and aggregate output for the period 1970-1998. Both series present almost the same high volatility captured by their respective standard deviation of 5.26% and 5.57% respectively. Thus, the Brazilian empirical evidence shows that the volatility of aggregate non-durable consumption series accounts for 94.43% of the volatility of aggregate output.

FIGURE 1 - CONSUMPTION OF NON-DURABLES AND GNP CYCLES (1970-1998)[5]



5 Source: ELLERY JR.; GOMES & SACHSIDA (2000). Filtered aggregate non-durable consumption series (H-P Filter, l = 100).

Simulations using the standard business cycle model generally reproduce a volatility of consumption as high as two third of the GNP series, well below the evidence shown by the data.[6] Clearly this point has to be theoretically better considered, in particular, if a business cycle model is used to analyze the Brazilian stylized facts.

A possible explanation, for the limitation of the standard business cycle model to account for the high volatility presented in the Brazilian consumption data, is based on the fact that a large proportion of individuals in the country face a liquidity (credit) constraint. Hence, they are not able to (optimally) smooth out their consumption stream over their lifetime. Therefore, these agents cannot behave according to Friedman (1957) and Hall (1978) permanent income hypothesis.

Issler and Rocha (2000) and Reis *et alii* (1999), among others, econometrically studied the Brazilian consumption series. The former authors show that consumption and output series have a common stochastic trend, for both series are co-integrated, and a common cycle, for variations in the expected value of consumption is proportional to variations in aggregate output. Moreover, they show that the tested model cannot be statistically accepted in its unrestricted form and, they estimate the share of liquidity constrained population to account for nearly 75% of the Brazilian consumers. The later, on the other hand, shows that the share of non-constrained consumers is statistically near zero. These studies strongly suggest the relevance of the large proportion of liquidity constrained agents behind the Brazilian aggregate consumption behavior, specially if these results are compared with Vahid and Engle's (1993) result estimating the proportion of USA consumers facing credit restrictions to account for 51% of them.

Another indication of the existence of such a large set of credit restricted agents in the Brazilian economy arises from econometric studies which

6 See COOLEY & PRESCOTT (1995), ELLERY JR.; GOMES & SACHSIDA (2002), KANCZUK & FARIA (2000) and VAL (1999).

try to estimate the value of the inter-temporal elasticity of substitution for the country. Cavalcanti (1993) and Gleizer (1991) suggest that this parameter, estimated from the response of the consumption to variations in the interest rate, has a value close to zero. These studies support the relevance of the liquidity constraint: the consumption of agents facing it will not be sensitive to variations in the interest rate.

Therefore, based on the above arguments, an extended model economy which accounts for a set of credit restricted agents will be built aiming to reproduce the high volatility of consumption present in the behavior of the Brazilian business cycle.

2. THE MODEL

The model economy consists of a continuum of individuals with measure one. Some of the agents are restricted to consume all their income at every period. Type 1 agents are non-restricted and λ measure their participation in total population, whereas type 2 agents face a complete credit restriction preventing them to inter-temporarily transfer their income and, $(1 - \lambda)$ represents the proportion of the later. There is only one non-storable good that may be produced in the economy with two alternative technologies.

The first one uses labor as well as capital input and is available only to the non-restricted (type 1) agents. Since the restricted (type 2) agents are not allowed to accumulate capital they are constrained to use an alternative technology. This alternative technology uses only labor to produce the single good of the economy. Moreover, assuming an additional hypothesis about the relationship among the parameters characterizing the inter-temporal preference and the technology available in the model economy, it will be optimal for the type 1 agent to use the technology only available to this type which is not available for the type 2 agent for this type does not have the required capital input needed. Therefore, each type uses only the respective available technology, to produce the single good.

An alternative setup would consist to assume that both types of agents use a common technology to produce the single good. This alternative would impose the need to calibrate the elasticity of substitution between the labor inputs provided for each type of agent. The calibration of this parameter rises the need to distinguish the agents on the basis of human capital, or any other observable characteristic that could affect the marginal productivity of labor. By doing so, the model would end up dealing with the problem of wage inequalities in Brazil that would have to be somehow incorporated into our model economy. This question generated a vast literature in the field, but the scope of wage differential goes far beyond the purpose of the present study. The herein used assumption on two types of technologies available to the agents allows avoiding those labor market considerations. Since the aim of the paper is to study the dynamic behavior of aggregate consumption[7] this modeling approach does not represent a problem whatsoever. The remainder of this section will describe the problem each type of individual faces, the technologies available to each of them, and the definition of recursive equilibrium of the model economy.

2.1 Type 1 Representative Agent (RA-1)

The agent of the first type is modeled as in the standard real business cycle model. At each period he/she chooses how much to consume and how much to save for next period. They are not subject to any restriction on their ability to borrow or to lend, so they will behave optimally in order to smooth out their consumption stream over the business cycle.

7 One of the referees has pointed that the a model with one technology would not be able to replicate the high volatility of the consumption *vis-à-vis* the volatility of the output as the investment made by the unconstrained is completely linked with the GDP and the consumption of the unconstrained would always offset the consumption of the constrained. We thank the referee for pointing out this question.

2.1.1 RA-1's Preferences

The type 1 RA has preferences defined over the stochastic sequences of consumption c_t^1 and leisure $l_t^1 = 1 - h_t^1$, described by the particular utility function given below:

$$U^1 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[(1-a)\ln(c_t^1) + a\ln(1-h_t^1) \right] \right\} \tag{1}$$

where E_0 denotes the expectation, and β the discount factor, with β E(0, 1). The individual has one unit of time each period to divide between leisure and hours to work (h_t^1). Every period this agent faces the budget constraint given by:

$$c_t^1 + i_t \leq w_t^1 h_t^1 + r_t k_t \tag{2}$$

where i_t is the investment, r_t is the real interest rate, k_t is the stock of capital previously accumulated and, w_t^1 is the real wage for individuals of type 1.

Therefore, the problem of type 1 agent will be set as maximizing the expected discounted utility (1) subject to the per period budget constraint (2), given the linear law of motion for capital accumulation:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{3}$$

where δ is the rate of depreciation. The initial capital stock, K_0, is assumed to be known to the household. Once the preferences are fully characterized the technology available to these type 1 agents are described below.

2.1.2 RA-1's Technology

The technology available to type 1 agent is characterized by a constant return to scale Cobb-Douglas production function given by:

$$y_t^1 = F(k_t, h_t^1) = z^1 k_t^\theta (h_t^1)^{1-\theta} \quad (4)$$

where the optimal choice of labor (h_t^1) and capital (k_t) of this type are the inputs to produce their output y_t^1. The exogenous technology shock is capture by z_t^1 containing a stochastic component given by ω^1, such that $z_t^1 = e^{\omega_t^1}$, which follows a first order Markov process:

$$\omega_{t+1}^1 = \rho\omega_t^1 + \varepsilon_{t+1} \quad (5)$$

where $0 < \rho < 1$ represents the first order auto-correlation coefficient and, ε an independent and identically distributed random variable drawn from a normal distribution, zero mean and finite variance σ_ε^2.

The firm optimally chooses capital and labor so that their respective marginal products are equal to the real return of per unit input in competitive markets, according to the first order conditions for the firm's profit maximization problem, i.e.

$$r_t = z_t^1 \theta\, k_t^{\theta-1} (h_t^1)^{1-\theta} \overset{m.c.c}{=} z_t^1 \theta\, K_t^{\theta-1} (H_t^1)^{1-\theta} \quad (6)$$

is the return to the capital stock services and,

$$w_t = z_t^1 (1-\theta)\, k_t^\theta (h_t^1)^{-\theta} \overset{m.c.c}{=} z_t^1 (1-\theta) K_t^\theta (H_t^1)^{-\theta} \quad (6)$$

is the real wage for labor input, where *m.c.c.* stands for the introduction of inputs markets clearing conditions.

2.1.3 RA-1's Dynamic Programming Problem

Given the above set up of the model for type 1 agents regarding their preferences and available technology, the corresponding dynamic problem faced by the RA-1 can be described as:

$$V_t(z^1,k,K) = \max_{\{c_t^1,h_t^1\}} \left\{(1-a)\ln(c^1) + a\ln(1-h^1) + \beta EV_{t+1}(z^{1'},k',K')\right\} \tag{8}$$

subject to (2),(3) and (5), where $V^1(.)$ denotes the value function to the representative agent of type 1. Furthermore, the agents are subject to the law of motion for aggregate capital:

$$K' = (1-\delta)K + I(z,K) \tag{9}$$

given initial per capita aggregate capital stock, K_0 and initial productivity shock z_0^1.

With the above description of the dynamic programming problem for type 1 agents, the problem for the credit restricted agents, type 2, will be characterized next.

2.2 Type 2 Representative Agent (RA-2)

Type 2 agents are assumed to be restricted to consume all their income at every period. Therefore, since they are not allowed to save, they do not accumulate any kind of capital. However, they can also produce the single good of the economy using an alternative technology with only labor as input. Due to this restriction, this type of agents cannot behave according to the permanent income hypothesis, they rather behave as an extreme keynesian consumer.

2.2.1 RA-2's Preferences

Similarly to the agent of type 1, the type 2 representative agent has preferences defined over stochastic sequences of consumption c_t^2 and leisure $l_t^2 = 1 - h_t^2$, described by the utility function:

$$U^2 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[(1-a)\ln(c_t^2) + a\ln(1-h_t^2) \right] \right\} \quad (10)$$

As a result of imposing the credit restriction over this type of agent, the corresponding budget constraint will take the following specification:

$$c_t^2 \leq w_t^2 h_t^2 \quad (11)$$

The main implication of the above restriction, (11), is that type 2 agents will be bound to decide how to allocate his unit time available per period between leisure and work. In this sense, there is not, strictly speaking, an inter-temporal choice problem imposed to this type of individuals. They will choose the same amount of leisure no matter the period.

2.2.2 RA-2's Technology

The technology accessible to the RA-2 is also characterized by a constant return to scale in its (single) labor input.[8] For simplicity, this technology is described by a linear relationship, hence, the production function will be assumed to have the following functional form:

$$y_t^2 = G(h_t^2) = z^2 h_t^2 \quad (12)$$

8 To present an example of production without capital is always a hard task, when writing this paper we were thinking about the workers who make his/her living out of watching the cars in the streets and parking lots, or working as housekeepers.

where h_t^2 represents the type 2 labor input choice and, z_t^2 the productivity shock particular for this type, containing a stochastic component given by ω^2 such that $z_t^2 = e^{\omega_t^2}$ which follows a first order Markov process:

$$\omega_{t+1}^2 = \rho\omega_t^2 + \xi_{t+1} \tag{13}$$

whc $0 < \rho < 1$ represents the first order auto-correlation coefficient and, ξ an independent and identically distributed random variable drawn from a normal distribution, with zero mean and finite variance σ_ξ^2.

It is assumed that the productivity shocks for each type of agent, z^1 and z^2, have the same persistence, capture by the parameter ρ, however, they may differ in their innovations, ε and ξ. Moreover, it is assumed that the innovations ε_t and ξ_t are independent of each other.

Based on the above technological assumptions, the optimal choice for the firm operating this type 2 technology is given by the first order condition for its profit maximization, setting the return to the type 2 labor input equal to the corresponding per period marginal productivity shock, i.e.

$$w^2 = z_t^2 \tag{14}$$

2.2.3 RA-2's Dynamic Programming Problem

As already mentioned in the above section 2.2.1, there is not an actual dynamic programming problem posed to the individuals of type 2. In order to prove this assertion notice that the Bellman Equation associated with type 2 agents has the following form:

$$V_t(z^2) = \max_{\{c_t^2, h_t^2\}} \left\{(1-a)\ln(c^2) + a\ln(1-h^2) + \beta EV_{t+1}(z^{2'})\right\} \tag{15}$$

subject to (11) and (12). It is straightforward to see that the solution to this problem will be:

$$h_t^2 = (1-a) \quad \forall t \tag{16}$$

which is clearly constant for every period.

Given the above set up for both types of agents of the model economy, the following additional assumption will be introduce in order to justify the (optimal) utilization of the first type of technology by type 1 agents, so that these agents will not have any incentive to use the technology with labor as the only production input.[9]

Additional Assumption: *The set of parameters characterizing the intertemporal preference and the available technology in the model economy are such that:*

$$\beta > \frac{1}{\theta(1-\theta)^{(1-\theta)/\theta} + 1 - \delta}$$

With this assumption there is no labor or capital market between the two types of agents for they only interact trough the market for the consumption good.

2.3 *Recursive Competitive Equilibrium*

A recursive competitive equilibrium for the above model economy consists of a set of value functions, $V^1(z^1,k,K)$ and $V^2(z^2)$; a set of decision rules, $c^1(z^1,k,K)$, $h^1(z^1,k,K)$, $i(z^1,k,K)$, $c^2(z^2)$ and $h^2(z^2)$ for the two types of individuals; a corresponding set of aggregate per capita decision rules, $C^1(z^1,K)$, $H^1(z^1,K)$, $I(z^1,K)$, $C^2(z^2)$ and $H^2(z^2)$; and factor price functions, $w^1(z^1,K)$, $w^2(z^2)$ and $r(z^1,K)$, such that these functions satisfy:

9 Appendix A shows that this additional assumption will be sufficient for the type 1 agent to optimally use the technology only available to him/her.

1. The dynamic program of the individuals of type 1, (8):

$$V_t(z^1,k,K) = \max_{\{c_t^1,h_t^1\}} \left\{(1-a)\ln(c^1) + a\ln(1-h^1) + \beta EV_{t+1}(z^{1'},k',K')\right\}$$

subject to (2), (3) and (5);

2. The dynamic program of the individuals of type 2,[10] (15):

$$V_t(z^2) = \max_{\{c_t^2,h_t^2\}} \left\{(1-a)\ln(c^2) + a\ln(1-h^2) + \beta EV_{t+1}(z^{2'})\right\}$$

subject to (11) and (13);

3. The optimal behavior of the firms of type 1, (6) and (7);
4. The optimal behavior of the firms of type 2, (14);
5. The consistence of individual and aggregate decisions, that is, the conditions:

$$c^1(z^1,K,K) = C^1(z^1,K), h^1(z^1,K,K) = H^1(z^1,K), i(z^1,K,K) = I(z^1,K),$$

$$c^2(z^2) = C^2(z^2),\ h^2(z^2) = H^2(z^2),\ \forall\ (z^1,z^2,K);$$

6. The aggregate resource constraint:

$$\lambda\left[C^1(z^1,K) + I(z^1,K)\right] + (1-\lambda)C^2(z^2) = \lambda y^1(z^1,K,H^1) + (1-\lambda)y^2(z^2,H^2)$$

This completes the description of the modeled environment and of the equilibrium concept. The next section will proceed with the calibration of the model economy and the results of the numerical simulations.

10 Observe that in fact, as discussed in the previous section, for this type, there is no, strictly speaking, a dynamic programming problem.

3. *CALIBRATION AND SIMULATION RESULTS*

This section will present the calibration for the model economy above as well as the main results from the simulations. The numerical method used to solve for the defined recursive competitive equilibrium was the so called eigenvalue-eigenvector decomposition.[11]

3.1 *Calibration*

The calibration of the parameters will be made based on the methodology proposed by Prescott and Cooley (1995). Therefore, the model is calibrated so that it matches certain long-run properties of the Brazilian data. However, for some parameter values, estimations or evidences from other authors will be used as it is the case for the intertemporal elasticity of substitution, the elasticity of substitution between consumption and leisure and the share of restricted individuals in the total population parameters' values. For the specification of the utility function it was assumed that both elasticities are equal to one. The lack of a consensual estimation for those parameters is the main reason to justify this choice. On the other hand, the lack of any trend for the per-capita leisure series suggests that the elasticity of substitution between consumption and leisure is near one.[12]

The rate of depreciation of capital, *d*, is set equal to 10%. For the share of capital income on aggregate output, the Brazilian National Account Statistics suggests that this share reaches 49% of the output. However, Gomes, Bugarin and Ellery Jr. (2001) argues that this number should be .33 due to the fact that the National Account System underestimate the labor income in Brazil. Following this observation implies that:

$$\frac{\lambda \theta y^1}{\lambda y^1 + (1-\lambda) y^2} = 0.33 \tag{17}$$

11 A fair description of the method may be found in FARMER (1993) and NOVALES *et alii* (1999).

12 See ELLERY JR.; GOMES & SACHSIDA (2002).

Observing that the value of the discount factor, b, should be able to reproduce the capital/output ratio observed in the Brazilian economy, which according to the series provided by the Institute of Applied Economic Research (IPEA) accounts to 2.7,[13] this share requires:[14]

$$\frac{\lambda k}{\lambda y^1 + (1-\lambda) y^2} = 2.7 \tag{18}$$

Data from various household surveys (IBGE-PNAD) suggest that the typical Brazilian spends nearly 31% of his time at work,[15] implying that:

$$\lambda h^1 + (1-\lambda) h^2 = 0.31 \tag{19}$$

The above calibration equations (17), (18) and (19) depends on the value of the share of the non-restricted agents in the economy, λ.[16] Following Issler and Rocha (2000) the proportion of unrestricted agents in the model economy, λ, will be set at 0.25.[17] With this parameter value at hand, it is possible to numerically solve the calibration equations (17), (18) and (19) for the set of parameters. Thus, the discount factor taken as $\beta = 0.98$, the share of capital income in sector 1 output $\theta = 0.53$ and, the marginal utility of leisure $a = 0.68$, turns out to be compatible with the empirical facts present in the Brazilian economy.

13 See IPEADATA (http://www.ipeadata.gov.br).

14 Actually, 2.7 represents the wealth/income ratio, this ratio seems to be more appropriated since real business cycle models assumes the stock of non-durables consumption good as a part of the capital stock.

15 See ELLERY JR.; GOMES & SACHSIDA (2002).

16 This share was estimated by ISSLER & ROCHA (2000) and by REIS *et alii* (1999).

17 This the estimated share of non-restricted agents in the economy, the consumption of these agents amount to nearly 50% of the total consumption of the economy as the income of these agents accounts for 62% of the total income of the economy.

Now it is only left the parameter values that describe the law of motion for the productivity shocks for the two types of technologies, z^1 and z^2. There are three parameters that are related with the stochastic shocks. The first is related with the persistence of the shock, ρ, its value was fixed as 0.589 as suggested by Ellery Jr., Gomes and Sachsida (2002). The other two parameters, δ_ε and δ_ξ, are related with the volatility of the shock. The value of δ_ε was set to 0.0446 as suggested in Ellery Jr, Gomes and Sachsida (2002), the value of δ_ξ was set to .0825. Table 1 below summarizes the calibration of the artificial economy.

TABLE 1 – CALIBRATION

Parameter	Value
Depreciation (δ)	.12
Capital share in sector 1 (θ)	.65
Proportion of unrestricted agents (λ)	.25
Discount factor (β)	.96
Marginal utility of leisure (a)	.68
Persistence of the technological shock (ρ)	.589
Standard Deviation of the Innovation 1 (σ_ε)	.0446
Standard Deviation of the Innovation 2 (σ_ξ)	.103

Even though the standard model was calibrated to reproduce the set of empirical facts present in the Brazilian economy, since all the output is produced in the non-restricted sector, the value of the parameter θ need to be calibrated again. To be consistent with the Brazilian National Account Tables this parameter should be set at $\theta = 0.49$. The other parameters will remain at their previous values. A small adjustment was made in the standard deviation of the innovation in order to allow comparisons between the model with liquidity constraints and the standard real business cycle model and, to reproduce the observed volatility of the Brazilian aggregate output, the standard deviation of the innovation process corresponding to the type 1 technology shock was set to $\sigma_\varepsilon = 0.044$.

Furthermore, the above calibrated parameter values are consistent with the additional analytical assumption, for $\beta = 0.96 > \dfrac{1}{\theta(1-\theta)^{(1-\theta)/\theta} + 1 - \delta} = 0.8$.

3.2 Simulation Results

Table 2 reports the statistical properties of the series generated by the standard growth model, the model proposed in this paper as well as the empirical evidences from the Brazilian data.[18] Focus will be given on the standard deviations of selected variables and on the correlation of these (current and one period lagged) variables with the output.

TABLE 2 - CYCLICAL BEHAVIOR FROM DATA AND SIMULATIONS

Brazilian Facts (1970 – 1998)			Correlations		
	σ_x %	σ_x / σ_{GNP}	(x_{-1}, GNP)	(x, GNP)	(x_{+1}, GNP)
Output	5.57	1.000	.5829	1.000	.5385
Consumption	5.26	.9443	.5487	.7540	.2402
Investment	12.77	2.293	.3579	.8510	.6509
Hours (from FIESP 1975 – 1998)	7.31	1.323	.3839	.6966	.4671
Standard Model					
Output	5.57	1.000	.3592	1.000	.3592
Consumption	2.03	.3650	.0790	.7725	.6889
Investment	13.20	2.370	.4110	.9830	.2370
Hours	3.09	.5560	.4380	.9528	.1453
Liquidity Constraints Model					
Output	5.57	1.000	.1541	1.000	.1541
Consumption	5.42	.9719	-.1265	.7708	.1375
Investment	11.12	1.993	.3633	.7609	.0974
Hours	.89	.1610	.7335	.7335	.0298

18 The facts were reported in GOMES, ELLERY JR. & SACHSIDA (2000).

If the Statistics derived from the Brazilian data are compared to the ones obtained from a standard real business cycle model, these numbers suggest that the standard model fails to explain the high volatility of the consumption[19] *vis-à-vis* the volatility of the output. In this sense, the present model with liquidity constraint provides a better match with the empirical facts in regard to the volatility of the consumption series than the standard model. Particularly relevant is the fact that the share of restricted agents proposed by Issler and Rocha (2000) is compatible with the model.

The contemporaneous correlation of consumption and output increases to a level above the observed in the Brazilian data. This is the case because more than 70% percent of the population is restricted to consume all their income at each period, meaning that for these agents the correlation between consumption and output is one.

The model assumes that a large share of the population operates a technology that uses only a labor input. So the model is prone to underestimate the correlation between investment and output. This effect is also illustrated in Table 1. The introduction of capital in the technology operated by type 2 agents should be able to reduce this undesired side effect, constituting a topic for further future study. As for the volatility of the investment series, the model with liquidity constraint as well as the standard model provides a fair match.

Table 2 also illustrates the inability of the model to account for the volatility of the worked hours. This is a consequence of using a Cobb-Douglas specification for the utility function.[20] To change the functional form of the utility function one should provide a set of empirical facts to support the new functional form.[21] Since the paper deals with the volatility

19 The series of consumption to the actual economy includes only the consumption of non-durables, see GOMES, ELLERY JR. & SACHSIDA (2000).

20 The authors thank an anonymous referee for this comment.

21 ELLERY JR.; GOMES & SACHSIDA (2002) provide evidence favoring the use of a Cobb-Douglas representation for the utility function in Brazil.

of the consumption series and not with the volatility of the worked hours we choose to keep the Cobb-Douglas functional form.

CONCLUSION

The paper proposed a model to explain the high volatility of the consumption *vis-à-vis* the volatility of the output observed in the Brazilian business cycle. Following the suggestion of Campbell and Mankiw (1989) it is assumed that a share of the population is restricted to consume all their income at each period. The structure of the model resembles the one suggested by Carmichael, Kéita and Sansom (1999) in the sense that both models present a dynamic general equilibrium setup with a subset of agents restricted to consume all their income in each period.

By incorporating this feature, the model is able to account for the large fluctuation observed in the consumption series relative to the output series, in the actual business cycle. The set up of the model is such that the credit constrained agents are restricted to operate a technology that uses only labor input. This assumption was introduced in order to allow the calibration as well as the numeric simulations of the model economy to be consistent with the empirical Brazilian aggregate business cycle evidences.

The model succeeds in explaining the high volatility of the aggregate consumption series, highly correlated with the aggregate production series observed in the actual data. However, it causes side effects on the series of investment, hours worked and on the lagged correlations as well, due to the assumption that the technology available to the type 2 agents uses only labor as input. The introduction of capital in the technology operated by the agents of type 2 is a suggestion for future research.

Furthermore, the numerical simulations of the model economy turned out to be fully consistent with previous estimates of the share of liquidity restricted consumers in Brazil. As econometrically tested by Reis *et alii* (1999) and Issler and Rocha (2000), the general equilibrium recursive model supports the hypothesis that nearly 75% of the consumers in Brazil are restricted to consume all their income at each period. An alternative approach where a share of the population will (optimally) choose to consume all their income at each period is another topic for future research.

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APPENDIX - SUFFICIENCY OF THE ADDITIONAL ASSUMPTION

The cost minimization problem for the type 1 firm can be written as:

$$\begin{aligned} &\min_{k_t, h_t^1} \; r_t k_t + w_t^1 h_t^1 \\ &s.t. \; k_t^{\theta} (h_t^1)^{1-\theta} = y_t^1 \end{aligned}$$

with the associated Lagrangean:

$$L = r_t k_t + w_t^1 h_t^1 + \mu \left(y_t^1 - k_t^{\theta} (h_t^1)^{1-\theta} \right)$$

the first order conditions (F.O.C) are:

$$\begin{aligned} &r_t - \mu \theta k_t^{\theta-1} (h_t^1)^{1-\theta} = 0 \\ &w_t^1 - \mu (1-\theta) k_t^{\theta} (h_t^1)^{-\theta} = 0 \\ &y_t^1 - k_t^{\theta} (h_t^1)^{1-\theta} = 0 \end{aligned}$$

From the firsts two equations of the F.O.C. one may obtain:

$$\frac{r_t}{w_t^1} = \frac{\theta k_t^{\theta-1} (h_t^1)^{1-\theta}}{(1-\theta) k_t^{\theta} (h_t^1)^{-\theta}} \Rightarrow \frac{r_t}{w_t^1} = \frac{\theta h_t^1}{(1-\theta) k_t} \Rightarrow k_t = \frac{w_t^1}{r_t} \frac{\theta}{1-\theta} h_t^1$$

substituting this result into the third F.O.C equation:

$$y_t^1 = \left(\frac{w_t^1}{r_t} \frac{\theta}{1-\theta} \right)^{\theta} (h_t^1)^{\theta} (h_t^1)^{1-\theta} \Rightarrow h_t^1 = \left(\frac{r_t}{w_t^1} \frac{1-\theta}{\theta} \right)^{\theta} y_t^1 \Rightarrow k_t = \left(\frac{w_t^1}{r_t} \frac{\theta}{1-\theta} \right) \left(\frac{r_t}{w_t^1} \frac{1-\theta}{\theta} \right)^{\theta} y_t^1$$

Therefore the cost function takes the form:

$$c_t^1(y_t^1) = r_t \left(\frac{w_t^1}{r_t} \frac{\theta}{1-\theta} \right) \left(\frac{r_t}{w_t^1} \frac{1-\theta}{\theta} \right)^{\theta} y_t^1 + w_t^1 \left(\frac{r_t}{w_t^1} \frac{1-\theta}{\theta} \right)^{\theta} y_t^1$$

while the marginal cost may be write as:

$$c_t^{1'}(y_t^1) = r_t\left(\frac{w_t^1}{r_t}\frac{\theta}{1-\theta}\right)\left(\frac{r_t}{w_t^1}\frac{1-\theta}{\theta}\right)^{\theta} + w_t^1\left(\frac{r_t}{w_t^1}\frac{1-\theta}{\theta}\right)^{\theta} =$$

$$= \left(w_t^1\frac{\theta}{1-\theta} + w_t^1\right)\left(\frac{r_t}{w_t^1}\frac{1-\theta}{\theta}\right)^{\theta} = \left(\frac{\theta w_t^1 + w_t^1 - \theta w_t^1}{1-\theta}\right)\left(\frac{r_t}{w_t^1}\frac{1-\theta}{\theta}\right)^{\theta} =$$

$$= \frac{w_t^1}{1-\theta}\left(\frac{r_t}{w_t^1}\frac{1-\theta}{\theta}\right)^{\theta}$$

Competitive input market implies that their returns must equal their respective marginal cost for both type of technologies. Moreover, those returns must be the same regardless the technology used to produce the good. Hence, the real wage of the type 2 agent must be equal to one[22] and, for the type 1 technology the condition $c_t^{1'}(y_t^1) = p = 1$, implies:

$$\frac{w_t^1}{1-\theta}\left(\frac{r_t}{w_t^1}\frac{1-\theta}{\theta}\right)^{\theta} = 1 \Rightarrow (w_t^1)^{1-\theta}\left(\frac{1-\theta}{\theta}r_t\right)^{\theta} = 1-\theta$$

$$\Rightarrow (w_t^1)^{1-\theta} = \left(\frac{\theta}{1-\theta}\frac{1}{r_t}\right)^{\theta}(1-\theta) \Rightarrow w_t^1 = (1-\theta)^{1/(1-\theta)}\left(\frac{1}{r_t}\frac{\theta}{1-\theta}\right)^{\frac{\theta}{1-\theta}}$$

$$\Rightarrow w_t^1 = (1-\theta)^{\frac{1}{1-\theta}} r_t^{\frac{-\theta}{1-\theta}} \theta^{\frac{\theta}{1-\theta}} (1-\theta)^{\frac{-\theta}{1-\theta}} = r_t^{\frac{-\theta}{1-\theta}} \theta^{\frac{\theta}{1-\theta}} (1-\theta)$$

$$\Rightarrow w_t^1 = \left(\frac{\theta}{r_t}\right)^{\frac{\theta}{1-\theta}}(1-\theta)$$

22 Remember that $E[z^2]=1$ and $y_t^2 = z_2^2 h_t^2$.

Thus, for type 1 agent will be optimal to work with the technology which uses labor and capital as long as $w_t^1 > 1$, i.e.:

$$\left(\frac{\theta}{r_t}\right)^{\frac{\theta}{1-\theta}}(1-\theta) > 1 \Leftrightarrow \frac{\theta}{r_t} > \left(\frac{1}{1-\theta}\right)^{\frac{1-\theta}{\theta}} \Leftrightarrow r_t < \theta(1-\theta)^{\frac{1-\theta}{\theta}}$$

But it is a well know fact that, in steady state one must have:

$$r_t = \frac{1}{\beta} - 1 + \delta$$

Introducing the steady state condition, the sufficient condition for the (optimal) utilization of the first technology by the type 1 agent becomes:

$$\frac{1}{\beta} - 1 + \delta < \theta(1-\theta)^{\frac{1-\theta}{\theta}} \Leftrightarrow \frac{1}{\beta} < \theta(1-\theta)^{\frac{1-\theta}{\theta}} + 1 - \delta$$

which also can be write as:

$$\beta > \frac{1}{\theta(1-\theta)^{(1-\theta)/\theta} + 1 - \delta}$$

This paper has benefited from comments on earlier drafts by Fabio Kanczuk, Victor Gomes, Marco A. Martins and Mauricio Bugarin as well as workshop participants at Universidade de São Paulo (USP) and Universidade de Brasília (UnB). We also thank the comments of two anonymous referees.

(Recebido em janeiro de 2002. Aceito para publicação em junho de 2002).