Constraints of learning mathematics: a systemic historical review of the literature[I]

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Abstract

In addition to the natural language, human beings hold an intuitive sense of counting. We have the ability to determine the number of objects in a small collection and carry out simple additions and subtractions without direct instruction. By the age of ten, a child understands about 10,000 words and speaks its native language with 95% accuracy. However, by the age of eleven, some children already claim that they do not understand mathematics. Why is that so? One reason is that spoken language and number sense are survival skills but abstract mathematics is not. This article presents a review of literature on the teaching of mathematics. Which web of interactions is established in the teaching-learning process of this subject? How much of what has been said and written about this process is merely a myth? This article aims at contributing in some way to the demystification and improvement of success in mathematics. It is, therefore, essential to understand the structural framework of theoretical research on the various approaches on the term learning and specifically on the difficulties in learning mathematics. This implies reviewing a set of internal (brain function, spoken language and learning style) and external constraints (socio-cultural factors and teaching styles).

Keywords

Mathematics – Learning styles – Socio-cultural factors – Brain function.

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Condicionantes da aprendizagem da matemática: *uma revisão sistêmica da literatura*

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Resumo

Para além da linguagem natural, o ser humano nasce com o sentido inato de número. Temos a capacidade de determinar o número de objetos de uma pequena coleção, de contar e de fazer adições e subtrações simples sem necessidade de instrução direta. Por volta dos dez anos, uma criança compreende cerca de 10.000 palavras e fala a sua língua materna com 95% de precisão. Contudo, por volta dos onze anos, algumas crianças já afirmam não conseguir compreender matemática. Por que essa diferença? Uma razão é que a linguagem falada e o sentido de número são capacidades de sobrevivência, mas a matemática abstrata não o é. Ao discutir os condicionantes do ensino dessa área do conhecimento, este artigo consiste em uma revisão bibliográfica acerca dos problemas do ensino da matemática. Que teia de interações se estabelece no processo de ensino-aprendizagem dessa disciplina? Quanto do que se tem dito e escrito a respeito desse processo não passa de preconceito ou mito? Com este artigo, pretende-se contribuir, de algum modo, para a desmistificação e a melhoria no sucesso da disciplina de matemática. Para tanto, é fundamental compreender o enquadramento estruturante da investigação teórica acerca de diferentes abordagens do conceito de aprendizagem e do que poderá estar em causa, especificamente, nas dificuldades da aprendizagem da matemática. Isso significa rever um conjunto de condicionantes internos (funcionamento do cérebro, língua falada e estilo de aprendizagem) e condicionantes externos (fatores socioculturais e estilos de ensino).

Palavras-chave

Matemática – Estilos de aprendizagem – Fatores socioculturais – Funcionamento do cérebro.

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Introduction

Mathematics has always been a key field in all education systems. It is an ancient science that has been taught as a mandatory subject of different grades for many centuries and has played a major role as a criterion for social selection. It is considered an absolute language, an infallible standard of knowledge, the key to progress. Other sciences, that have allowed us to understand the mysteries of mankind, of nature, the world and the universe, feed themselves, to a great extent, from Mathematics. Its importance comes from afar and goes further. In the fifth and sixth centuries BC, Pythagoras and his followers believed that mother nature has a mathematical underground and "numbers rule the world", and Plato, to whom the statement "God geometrizes always" was attributed, is said to have placed a sign on the top of the doors of his school facilities with the well-known sentence "Let no one ignorant of geometry enter". Napoleon already invoked mathematics to legitimatize power relations by stating that "men are like numbers, their value depends only on their position". Despite the importance associated with these facts, mathematics has been considered a difficult learning field throughout the years.

It is a fact that mathematics is seen by a high number of students as a difficult subject that deals with extremely abstract, more or less unintelligible objects and theories. However, how much of the students' opinions is intrinsic to their real experience and not only a result of reduplicated utterances from other voices, as echoes that were heard from their parents, friends, social media, and even teachers? How much of that speech does not bring another underlying speech, a pre-built speech that is engraved in their memory? Education critics consider that only a small number of students are really unable, in terms of development, to deal with mathematics and that, globally, the poor performance in this field is due mainly to an unsuitable teaching approach. One thing seems certain: students who are weak in mathematics in the early years remain weak in their later years (SOUSA, 2008).

A complete research project of the intercultural learning of mathematics cannot be confined only to cognitive aspects (understanding, reasoning, problem-solving, etc.); thus, an analysis of external factors is imperative in its theorization and empirical research. From times immemorial, the teacher has been considered as the driver of students' learning, and what they learned depended on what and how the teacher taught, as if teaching and learning were an independent and one-way relationship. Nowadays, the teaching-learning process refers increasingly to a clear assumption that efficiency and effectiveness of teaching are not unrelated to the understanding of how learning is processed in the students.

Which interactions are established in the teaching-learning process of mathematics? Which factors influence positively or negatively this learning process? The complex web of interactions that takes place in the learning process of mathematics is interwoven with a great multitude of wires, aspects to be considered. In general, several constraints might be taken into account, both internal and external to the students. The functioning of the brain (Section 2.1), the spoken language (Section 2.2), and learning style (Section 2.3) can be regarded as the major internal constraints. Socio-cultural factors (Section 3.1) and teaching styles (Section 3.2) are assumed to be external ones. Certainly, the boundaries between and within these two types of limitation are not sharp; on the contrary, they interact, build on each other and fall apart continuously and dialectically.

Internal constraint: brain functioning

What is knowledge, how is it processed, what is the role of the brain and of the spoken language in the learning process in general and in mathematics in particular? (PIAGET, 1954) scientifically approaches some questions of the knowledge theory through the genesis of the cognitive subject structures. He distinguishes formal from empirical knowledge and, taking into consideration the origin of these both types of knowledge, Piaget sidesteps the rationalist, empiricist, and traditional views. For the empiricist, the origin of knowledge leans on reality, whilst the subject's mind represents the passive container of knowledge. For the rationalist, knowledge is innate and its evolution is solely an update of pre-built internal structures. According to Piaget (1970), who introduced the notion of constructivism, knowledge construction implies the interaction between the subject that knows and the known object; it is the subject itself that, driven by the action, builds his own representation of the reality, by interacting with the object of knowledge. For Piaget, the acquisition of knowledge occurs only with the consolidation of mental structures that can be divided into four stages: sensorimotor, preoperational, concrete operational, and formal operational.

Although introduced by Piaget, the concept of constructivism, according to Castañon (2009), was developed in several study areas, from mathematics to logic, from psychology to sociology, from education to psychotherapy and even in the neurosciences. The major contemporary streams of this conduct line are the following:

• Radical Constructivism, presupposing that knowledge is not more than a constant construction, and based on subjective information of our experience, is defended by theorists such as Ernest von Glasersfeld, Paul Watzlawick and Heinz von Foester.

• Logical Constructivism, better known as intuitionism – an approach to logic that emerged within the philosophy of mathematics – finds its major theoretical reference in Luitzen Brouwer; in this particular case, constructivism argues that mathematical objects are mental constructions that occur in a pre-linguistic mental structure.

• Social Constructionism (not social constructivism), which finds its major theoretical reference in Kenneth Gergen, and is based on three presumptions: Reality is dynamic and does not hold any essence or immutable laws; knowledge is only a social construction based on linguistic communities; knowledge has social consequences and these are the ones that should determine whether it is valid or not.

• Socio-constructivism or Social Constructivism – an approach developed out of the Lev Vygotsky's work – that explains human development as the social development of the child with the presupposition that knowledge is a social production and it is done through systems and strategies of social mediation-representation. The highest psychological functions are the result of cultural and not biological development.

Piaget's theory on cognitive development stages influenced many of the learning-teaching conceptions of mathematics, namely the idea that a child that has not yet reached the sensorimotor stage will not have the cognitive structures that allow him/her to, for example, understand the permanency of objects. According to Devlin (2000), referred to by Araújo (2006), this stage notion fell at first when "toddler's experiments with a couple of months showed that they had grasped, not only the notion of object permanency, as they also understood the basic notions of numbers by expressing object quantities". Number sense, according to Devlin (2000), consists of the ability to compare the sizes of two groups shown simultaneously and on the skill of recalling the number of objects shown successively. Although number sense is innate to the human being, it does not necessarily mean that all of us will become great mathematicians. Still, we can potentially be much better in arithmetic and mathematics than we may expect. Recent studies about number sense have undermined the theory of Piaget (SOUSA, 2008).

Another impact of the Piagetian theory, related to the notion of number conservation, has its root in the belief - widely spread in how to teach mathematics in the early school years - that to count and to memorize numbers would not be central, not even necessary for the construction of the number quantity notion. For Piaget (1954), that notion starts developing from logical thinking, between four and seven years of age, and after the notion of number conservation is then acquired (BUTTERWORTH, 1999 quoted by ARAÚJO, 2006). This idea was defeated by an experiment carried out by Mehler & Bever (1967), in which both researchers asked children to pick one of different-sized columns of Smarties or M&M candies. Sometimes, the column made up of four M&Ms was more widely spaced than the one with six M&Ms; at other times, the column consisting of six M&Ms was the most widely spaced one; some other times, both columns had the same length. In all conditions, even the two-year old children always chose the column with more M&Ms (ARAÚJO, 2006). Crato (2006) refers to the fact that the prestige of Piaget (and, to a less extent, of Kohlberg) allowed generations of teachers to be taught in a way to reduce their expectations towards their students, according to what the "psychological research" supposedly admitted to be possible in each age group or development stage.

The neurologist Castro-Caldas (2006), in his paper "Os processos neurobiológicos subjacentes ao conhecimento da matemática" [The neurobiological processes subjacent to the knowledge of mathematics], considers that memorization and understanding are complementary and not antagonistic; he cogitates that, from the biological point of view, memory training generates more neurons and links among them and, thus, gains future importance by stating that:

> [...] learning by heart, for example, all the names of rivers' streams generates internally an abstract memory that all rivers have streams, which facilitates the learning of streams of new rivers and this notion can be extended to the streets and paths and even to the knowledge of the body's arteries and blood vessels. We can conclude that these memorization processes generate complex matrixes for identification per analogy, which is the fastest way of processing information in an adaptive mechanism context. The more perfect and varied the matrixes available, the more effective will be the recognition and the abstract processing of information. (CASTRO-CALDAS, 2006, p. 196, own translation)

Yet, he draws attention to the fact that learning implies challenging the practice drills or repetition routine and, as such it is a factor for concern.

Neuroscience has advanced a great and recent step forward, which has influenced and changed the quality and quantity of information about the brain. With modern medical technologies it has become possible to observe the brain's activity and obtain knowledge of how the brain processes mathematical operations. It is still a mystery when and how human beings developed the ability to count beyond the innate sequence of "one, two, many". For Sousa (2008), maybe everything started in the same way children still do nowadays, i.e. by using their fingers. On one hand, our base-10 number system suggests that the counting process can be linked to the enumeration by using fingers; on the other, the word borrowed from Latin, digit, was used for the meaning of both numeral and finger. Typical brain scans support the idea of connections between numbers and fingers. When we do basic arithmetic, the highest brain activity is located in the left parietal lobe and in a part of the motor cortex that controls the fingers (DEHAENE et al, 2004). Some researchers speculate that human ancestors used fingers in their early experiences with numbers and that same part of the brain that controls the fingers becomes later, in their descendants, the area where the most abstract arithmetic activity is located (DEVLIN, 2000).

Internal constraint: spoken language

Language is an exclusive attribute of human beings, playing a major role in our experience and, subsequently, in our knowledge. Is language a mere expression tool of our thoughts or does it form thinking itself? Do people who speak different languages think in different ways only because they speak different languages? Does the learning of new languages change our way of thinking?

Studies of brain scanning images reveal that Chinese native speakers process arithmetic manipulations in different brain areas when compared with English native speakers, while researchers speculate that the biological code of numbers may differ in these cultures because their spoken languages are written in a different way, which results in dissimilar visual reading experiences (TANG et al, 2006).

On the other hand, Hans and Ginsburg, (2001) claim that language plays an essential role in the learning of mathematics, as it is by its means that ideas are expressed and mathematical concepts are defined, just as connections between different mathematical representations are conveyed. The impact of language as a cognitive tool can vary, with the Chinese language being a great example as a facilitator of learning mathematics.

The system of counting in Western languages causes more difficulties for children who are learning to count than for Asian children. Under western practices, it is more difficult to retain numbers in a short-term memory, which makes the learning to count and the perception of the base-10 numbering system more arduous, therefore creating delay in the calculation process. According to Sousa (2008), when we try to recall a list of numbers by saying them aloud, we are using our verbal memory, a section of the immediate memory that is able to retain information during two seconds. By doing so, the extension of our memory remains limited to the number of words we can pronounce in less than two seconds. It happens that the naming of Chinese numbers is much shorter than in most Western languages. The majority of Chinese numbers can be recited in less than a quarter of a second, while pronouncing them in English takes about one third of a second. Cantonese speakers have a memory extension for numbers of about ten digits, as opposed to the seven in Western language speakers, which make memorization easier for the former ones. Nevertheless and unlike Western languages, Chinese and Japanese hold a syntax number that facilitates its learning and memorization, perfectly reflecting the decimal framework. In Chinese, only eleven words are used to count the first one hundred numbers. In Portuguese (or English), twenty-eight words are needed. Under the Chinese arrangement, the naming of the first ten numbers is completely arbitrary, similar to other languages, but from ten onwards, one says ten one (for ‚eleven‘), ten two (for ‚twelve‘), etc.; from twenty onwards, one says two ten (for ‚twenty‘), two ten two (for ‚twenty-two‘), etc.; from thirty onwards, three ten (for ‚thirty‘), three ten one (for ‚thirty-one‘), three ten two (for ‚thirty-two‘) and so on till one hundred, from which the logical decomposition of numbers takes place (see Table 1).

As a result, Asian children start learning to count in average earlier and better than their Western counterparts (at around the age of four Chinese children are able to count up to forty while Western children, at the same age, can barely count to 15), and they start carrying out simple additions/subtractions earlier (32, three ten two, added to 27, two ten seven, is 59, five ten nine).

Apart from these aspects closely related to counting and operations with numbers, other mathematical queries become more understandable for Chinese native speakers. Western languages include often words stemming from Greek and Latin, whose interpretation is not immediate, such as the Portuguese word hexágono (in English, ‘hexagon’) that contains two elements stemming from Greek: hex (meaning ‘six’) and *gonia* (meaning ‘angle’). As to the Chinese language, the characters 六邊形

Table 1- Comparison of numbers in Portuguese vs. Chinese (Cantonese)

Al-Khowarizmi's Symbols	Reading in Portuguese	Chinese Characters	Reading in Cantonese
1	Um	一	yat
2	Dois	二	yi
3	Três	三	saam
4	Quatro	四	sei
5	Cinco	五	m
6	Seis	六	lôk
7	Sete	七	tsat
8	Oito	八	baat
9	Nove	九	gau
10	Dez	十	sap
11	Onze	十一	sap yat (ten one)
12	Doze	十二	sap yi (ten two)
13	Treze	十三	sap saam (ten three)
14	Catorze	十四	sap sei (ten four)
15	Quinze	十五	sap m (ten five)
16	Dezesseis	十六	sap lôk (ten six)
17	Dezessete	十七	sap tsat (ten seven)
18	Dezoito	十八	sap baat (ten eight)
19	Dezenove	十九	sap gau (ten nine)
20	Vinte	二十	yi sap (two ten)
21	Vinte e um	二十一	yi sap yat (two ten one)
...	...	...	...
30	Trinta	三十	saam sap (three ten)
40	Quarenta	四十	sei sap (four ten)
50	Cinquenta	五十	m sap (five ten)
60	Sessenta	六十	lôk sap (six ten)
70	Setenta	七十	tsat sap (seven ten)
80	Oitenta	八十	baat sap (eight ten)
90	Noventa	九十	gau sap (nine ten)
100	Cem	一百	baak
	Total of different words: 29		Total of different words: 11

Source: authors elaboration.

mean 'figure with six sides' (six bin shape). Milligan and Milligan (1983) carried out a glossary study about the mathematics teaching books written in English, which shows that 83% of all the terms contain Greek and Latin root words. On the contrary, the majority of mathematical terms in Chinese bears a descriptive nature and is conceptually clear, with a concrete and visual register (HAN; GINSBURG, 2001).

Internal constraint: learning styles

Several theories have been developed to describe the differences observed in the approaches to students' learning, among which the theory of multiple intelligences by Gardner (1993) and learning styles by Kolb (1981) stand out. Howard Gardner began his research in the 1970s on development and neuropsychology, culminating in the multiple intelligence theory in 1980 and bringing to an end the hitherto widely accepted idea of a single, general intelligence. His theory, according to which there are different human abilities that range from musical intelligence to the one that involves the understanding of oneself, was the subject of his work, *Frames of Mind: The Theory of Multiple Intelligences*, published in 1983. Gardner asserts the existence of eight effectively demonstrated intelligences: verbal-linguistic, logical-mathematical, visual-spatial, rhythmic-musical, bodily-kinesthetic, naturalistic, intrapersonal, and interpersonal. A ninth one, existential-moral intelligence, still needs deeper insight and revision in order to be added to the already accepted eight. Gardner points out eight criteria to be considered so that any behavior manifested in any individual can be regarded as intelligence:

• Intelligence identification and its respective location as a result of brain damage;
• Existence of idiot savants who reveal major limitations at certain intelligence levels and exceptional performance in some others, which allows us to observe them separately;
• Intelligence manifestation through occasional stimulus;
• Susceptibility to intelligence change through training;
• Intelligence presence in the evolution genesis of the humankind;
• Demonstrated intelligence via specific examinations that allow intelligence autonomy to be investigated;
• Support from psychometric findings;
• Existence of a specific symbolic system that enables intelligence isolation.

Another model for learning styles was developed by David Kolb in the 1980s. Kolb (1981, 1984) considers that interaction between concrete experience and theoretical conceptualization turns learning into a cyclic process, built up on four steps: concrete experience, reflexive observation, abstract conceptualization, and active experience. These stages are present in the following four learning styles, based on the dichotomies active/reflexive and abstract/concrete:

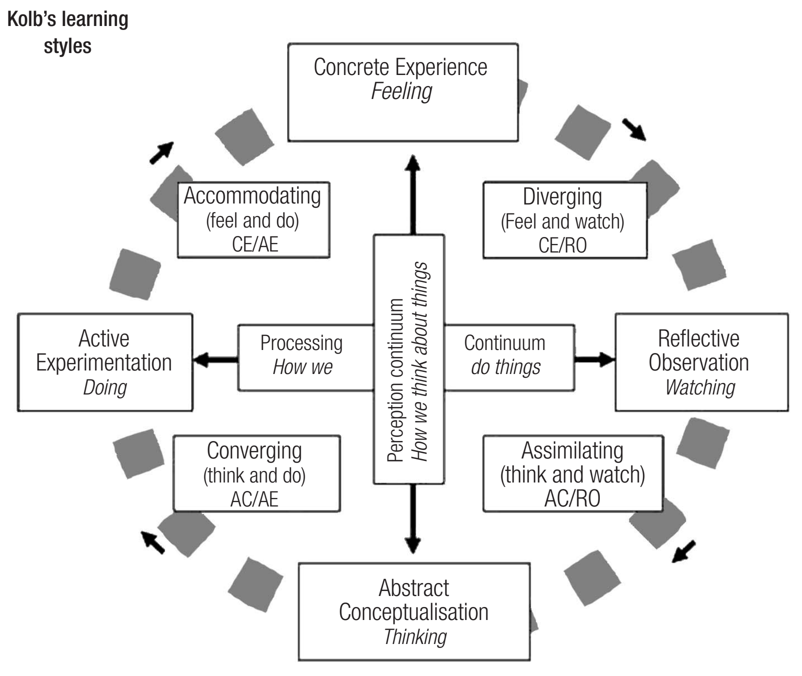
• Converging (thinking and doing), where abstract conceptualization and active experience predominate: This is the typical style of those who are more attracted to technical problems and solving tasks rather than interpersonal or social issues, quite often, presenting better skills in the practical application of ideas;
• Diverging (feeling and observing), featured by concrete experience and reflexive observations: This is the case of those who are sensitive and prefer to watch rather than to do, with a tendency to obtain information and use creativity to solve problems. By nature, they are creative, emotional and interested in people and group-work. These learners react well to brainstorming, making contributions for a better performance. It is highly connected to people who are strong in arts and human sciences;
• Assimilating (thinking and observing), where abstract conceptualization and reflexive

observation are dominant, by allowing these learners to be more interested in abstract ideas rather than in people. Understanding and creating models is one of these learners' strengths. Quite commonly, individuals who work in mathematics tend to have this learning style, just as those whose work involves planning and research;

• Accommodating (feeling and doing), whose personalities are stronger in concrete and active experiences: They trust their intuition, to the detriment of logics, i.e. they trust information given by other people rather than trusting their own analysis, preferring a practical and experimental approach, not being afraid of making mistakes in problem-solving tasks. They are risk takers.

The following diagram (see Figure 1) shows the mapping between the learning cycle and the learning styles.

Figure 1- Kolb's diagram of learning styles. According to this theory, every student, regardless of his/her learning style, can reach the same proficiency level through adequate methodologies and training processes.



Source: www.businessballs.com/freepdfmaterials/kolb_learning_styles_diagram_ colour.pdf, 2012.

Lilienfeld et al (2010) highlighted the problem that one could encounter if one reduces students' scholastic insuccess to the fact that teachers cannot adjust themselves to the students' learning style, and if one does not take into account the students' learning ability and motivation. Under this context, they also refer to a humourous article of a North American satirical newspaper, The Onion, entitled "Parents of nasal-learners demand odor-based curriculum", ridiculing the idea that there is a teaching style to unchain the unknown potential of all students with academic performance. They consider that the belief that stimulating teachers to adapt their teaching style to their students' learning styles can improve the learning process, is no more than an urban legend of educational psychology. The results can be the opposite of what we want to achieve when we focus teaching on students' strongest intellectual skills rather than on their weaknesses because learners need to correct and overcome their deficits instead of avoiding them. Moreover, life outside school is not always compatible with our favorite learning styles and good teaching is the one that prepares us for the real world.

External constraint: social-cultural factors

Recognized as one of the greatest historical personalities of Chinese civilization and culture, Confucius left a great mark in China. His teachings had a significant influence not only in the educational domain, but also in political, economic and cultural fields, and the ethical and moral domain. From the pedagogical perspective, his doctrine is most probably the oldest one that is still in place, with over 2500 years of accumulated experience and in a full-swing process of application, development and adaptation to present times. Many of Confucius', or to him attributed, sayings are still pertinent and of great relevance.

Associating Confucius with education is, therefore, inevitable, and those links are not limited to traditional Chinese education. His large contribution to education and teaching has given Confucius a very special place in history and culture. Sometimes idolized, sometimes despised, his legacy still influences one fifth of the world population. His pedagogy is still remarkable, while experts wonder how his ideas resist to time. Certainly, the universal validity of his thought and the fact that it addresses the needs and immediate and permanent problems of mankind are a plausible explanation.

Many Western observers point out the incongruence between the positive results that students of Asian countries achieve in mathematics and their obsolete teaching, based on mechanized and routine learning, not leading to the development of higher skills. A teaching system with an authoritarian, teacher-centered pedagogy and a centralized curriculum does not appropriate for differentiated educational needs and does not promote creativity, on one side, and its biddable, obedient and non-critical students, who learn by means of drills, lacking intrinsic motivation and having examinations as their final goal, on the other (GARDNER, 1989; GINSBERG, 1992; OUYANG, 2000). This is a teaching system where, even today, education is a social ladder, while the admission to universities is very competitive, such as it was till the beginning of the 20th century in China the selection of civil officials, which depended on the competitive imperial exams.

This "paradox of the Chinese student" represents, according to Biggs (1996), an apparent paradox, as there are values, deeply rooted in the Confucian culture and socialization practices, that increase the receptivity of those students to school learning and that are susceptible to deceiving Western observers. Children start very early to develop beliefs about learning based on their culture and those beliefs determine their learning and the outcomes they achieve. For centuries, the Chinese believed in the value of education either for collective benefit or individual development. Thus, the major significance of the education role has both a historical and a current meaning.

Leung (2001) systematized six dichotomies when comparing the teaching of mathematics in Asia and in Western cultures:

1) Outcome (contents) *versus* Process: Subjacent to this dichotomy, there are different points of view regarding mathematics. Is mathematics essentially the product (a body of knowledge) or a process (a single way of dealing with particular aspects of reality)? Although Western and Asian scholars assert that both aspects are components of mathematics, it is their position between both extremes of this continuum scale that differentiates them. Western scholars consider their Asian homologues outdated because they "stick" too much to content without following the trend in recent decades of focusing teaching-learning on

the process rather than on the outcome, namely through research activities and problem-solving tasks. Asian scholars believe that their Western homologues go too far in the valorization they assign to the process itself, and they underline the importance of content in the learning process of mathematics.

2) Memorization *versus* Significant Learning: While Western memorization is linked to mechanized and meaningless learning, Asians face it as legitimate, because it is considered to be an interactive process of a repeated practice, memorization and understanding, so that perceiving memorization as mere mechanized learning is a too simplistic point of view.

3) Hard Study *versus* Recreational Pedagogy (or edutainment): Western educators hold the opinion that it is crucial for students to enjoy themselves and have fun while they learn, whereas, from the Asian point of view, learning and study necessarily imply hard work; the pleasure of learning comes essentially from the results achieved. In this sense, (HUANG, 1969) refers to the sentence pinned on a school wall in Beijing: "the roots of knowledge are bitter, but its fruits are sweet".

4) Intrinsic *versus* Extrinsic Motivation: For Westerners, intrinsic motivation is overvalued and considered to be the best way of keeping students interested in studying mathematics, whereas the extrinsic incentive, such as preparation for exams, is solely a cause for anxiety, damaging the learning itself. Although Asian educators do not disagree on the relevance of intrinsic motivation, exams have been traditionally considered to be an acceptable motivation source for students' learning. They also consider that both can be complementary, although the distinction between both types of motivation is not clear.

5) Class *versus* Individualized Teaching: From the Western perspective, individualized teaching-learning is considered to be the ideal model, and the existence of students' classes is essentially justified for economic reasons and other resource limitations. In the Eastern culture, education is understood mainly as a socialization process; therefore, collective, class or group learning is highly valued. This difference leads to different understandings of the teacher's role. The main role of Western teachers is to meet individual needs of students and, on this basis, individualized syllabi and similar initiatives are the ideal way to teach and learn, preferably in classes that should be as small as possible.

In Eastern cultures, the teacher's role as a model is essential, so teaching and learning in a group makes sense, and group size does not represent a limitative factor. From this perspective, in China and in Asia in general, it is not class size that determines the quality of the teaching-learning process, but rather the teacher's qualifications and the way in which he leads that process. Leung (2001) considers that, most of the times, many syllabi for individualized learning merely degenerate into interactions between the student and (namely technological) learning materials, rather than between the student and the teacher, not providing the student with the opportunity to discuss, observe, listen to the teacher, i.e. to learn by having the teacher as a model.

6) Teachers' Scientific Competence *versus* Pedagogy: The "knowledge boom" and the easy access to the internet contributed to the belief that the teacher will not be able to compete with that knowledge potential, reducing his importance in the teaching-learning process. The teacher is less and less the source of knowledge, merely leading students to knowledge. He is a facilitator of learning, helping the students on how to learn, even when he does not master or sometimes does not know the content. This explains that the main concern has become the pedagogical competence question rather than the scientific competence. According to Leung (2001), this is an issue in Western countries with mathematics teachers at primary level. On the contrary, the image of the mathematics educators in Asian countries is still one of an "expert". As expected,

this scholar image (scholar teacher) has deep roots in the Confucian culture and still has a great impact on the education of those countries. This is confirmed by the Chinese proverb: "a teacher needs to have a bucket of water before he is able to give students a bowl of water". In the teaching of mathematics, this implies that, in spite of the fact that pedagogy is a major factor in the teaching-learning process, a good command of the program contents is even more important. This being said, the teacher has to be above all a scholar and then a facilitator of the learning process. The dichotomy between program contents and pedagogy suggests, therefore, the existence of different views on the teacher's role, and although in both cultures the teacher's scientific competence is considered to play a significant role, in the so-called Asian countries this assumption goes further, to the extent that it is not possible to be a teaching facilitator without being a scholar.

This position is supported by Tung (2000) who considers that without a strong knowledge of advanced mathematics it is difficult for a teacher to teach well, even at the basic level. (MA, 1999) goes further by asserting that without the knowledge of advanced mathematics it is not possible to make use of the appropriate pedagogy, not even at a basic level. Finally (QUEIRÓS, 1945), in his corrosive meditations on prominent recipes for teaching, holds the opinion that in order to be able to teach there is a tiny, little formality to be observed: knowledge. These particularities that derive from the cultural variables of the learning process, and their inaccurate interpretation, because they are analyzed in different cultural universes, might lead to abusive generalizations and to the constructions of expected outcomes that turn out not to be manifested.

External constraint: teaching styles

Ponte (1992) maintains that mathematical knowledge is based on four main features: formalization according to well-defined logics; verifiability that allows us to reach consensus on the validity of each result; universality, i.e., its trans-cultural character and the possibility of applying it to the most varied phenomena and situations; and, generativity, which relies on the possibility of discovery, being made up of four building blocks: basic skills (that imply processes of simple memorization and execution), intermediate skills (that involve processes to a certain level of complexity, but do not require a lot of creativity), complex skills (that entail a significant ability to deal with new situations), and knowledge in a broad sense (that include meta-knowledge, i.e. knowledge that affects the previous knowledge itself). The development of mathematic knowledge relies on action (manipulation of objects and of numerical, graphic and algebraic representations), and reflection (thinking over the action, stimulated by the tentative explanation and discussion). (PONTE, 1992) also considers that the conceptual substract plays a determinant role in the thought and in the action and, subsequently, in the teacher's practice.

The so-called traditional teaching styles are based on methodologies and assessment types that tend to prioritize the teacher's transmission of knowledge to students, who, in turn, should be able to reproduce what was conveyed to them, by turning teaching into a mechanized process, in which students are neither required to think nor to develop independent, creative thinking. There is no room for thinking over action, for reflection. This teaching is centered on content and the student takes on a passive role, merely concerned with memorizing content and recalling it when required to do so in equally traditional assessment practices, namely examinations and assessment tests. This is the teaching scheme that Freire (1975) named "banking concept of education", in which the teacher builds "deposits" that the learners patiently receive, by memorizing and repeating, and in which the only scope of action provided is to receive the deposits, file and store them. The knowledge mobilization in general, the meta-knowledge, hardly occurs. These features are believed to occasionally originate an out-of-context teaching, reflecting itself on the non-integration of the student in the society as a critical and participative agent.

In the context of the didactic practices of teachers – those that take place in the classroom – the discursive practices, due to the transversality of language in the human activity, have an important dimension. If the teacher's practice in classroom mirrors his teaching style, his speech will also reveal his position towards central issues related to mathematics teaching mentioned previously. In the Portuguese version of the Professional Standards for Teaching Mathematics by the National Council of Teachers of Mathematics (NCTM, 1994), it is stated that "the classroom speech mirrors what it means to know mathematics, what makes something become true or reasonable, and what implies to do mathematics; it is, therefore, of major relevance what students learn about mathematics just as much as how they learn it" (own translation).

Ernest (1988), cited by Thompson (1992), based on an empirical study, highlights three elements that can influence the teachers' practices: (1) The teacher's beliefs system on mathematics and on its teaching and learning; (2) The social context where its teaching takes place, particularly the obstacles and opportunities that it creates; (3) The teacher's thinking and reflection level.

What concerns the beliefs about mathematics, Thompsom (1992) argues that it is not a simple cause-effect relation, but rather a dialectic relation, extremely complex and of poorly defined contours. In turn, Ernest (1988) considers the following typologies of beliefs, regarding the way mathematics can be understood:

1) As a tool box (from an utilitarian perspective), in which mathematical knowledge is a set of facts, which are not necessarily related to each other, and serves the development of other sciences and techniques;

2) As a static and unified body of knowledge (from a platonic perspective) which implies that one can only discover but not create new knowledge;

3) As an open field to human creation that expands continuously, leading to new models and procedures (from a perspective of problem resolution).

Chacón (2003), following Thompson (1984), systematizes the teacher's role within the classroom according to each of those perspectives. In this sense, an instrumentalist teacher will have a teaching method that is rather prescriptive, by emphasizing rules and procedures. A platonic type will highlight the meaning of concepts and logics in the mathematical procedures. Finally, a teacher who is in line with problem resolution will give more importance to the exercises that lead students to become interested in the generative processes of mathematics. The teacher's role and intervention take on a significant importance, by being, in the first case, a mere instructor and, in the last case, a facilitator or mediator in the construction of students' mathematical knowledge.

Regardless of the connections that might be established between learning style and successful learning among students, the lecturer's role is irrefutable. The significance of the teacher's role is emphasized in the previous mentioned Professional Standards for the Teaching of Mathematics (NCTM), according to which teachers are the main protagonists of change by which mathematics is taught and learned at schools. In the same document, there is also a reference to the fact that teachers' – and students' – beliefs and conceptions build obstacles to major changes in the teaching and learning of mathematics at schools.

Final considerations

The unique nature of the human beings, on the one hand, and their cultural side, on the other, do not allow the teaching-learning process to be linear and plainly modeled. A more holistic vision and subsequent systemic approach of that process are mandatory. None of the above-mentioned constraints is per se an absolute factor of either academic success or failure. What seems to work well in some situations can have disastrous effects in others. It is essential to think and act "out of the box", without prejudice or pre-conceived ideas. A complex learning architecture presupposes a permanent design of adaptation, a continuing design and redesign of fashions, methods and styles in an endless search for the teacher's commitment to the students' learning. Which teaching design will better serve is an endless search. One thing seems to be certain: the improvement of mathematics teaching implies necessarily the non-overestimation of a single methodological line. On the contrary, it will have to be drawn from a process of diversified methodologies and be based on coherent psychosocial grounds.

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