ARTICLES

Reflections on the educational practice of active Panamanian mathematics teachers*[1]

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Abstract

The purpose of this document is to analyze the results of previous research where the components with the low or null presence of the Didactical Suitability Criteria (DSC) in developed tasks (didactic sequences) for a training course in the teaching of Mathematics are reviewed. Individual semi-structured interviews were conducted with five teachers who participated in the previous research to gather information on why these results were obtained. The interview consisted of eight questions on the component's connections, didactical innovation, representativeness of complexity, errors, social-professional practicality, and autonomy. The analysis was conducted through a qualitative study of the semantic units of meaning (descriptive analysis). These were extracted from the transcripts of the interviews guided. The results revealed that most interviewees indicated that components, such as autonomy and social-professional practicality, generate positive emotions in their students. It is concluded that the teachers' reflection on their mathematics classes up to that time was considered a non-priority for the components whose score was low. However, the interviewees considered the importance of modifying their educational practices by employing other components of the DSCs to have quality teaching and learning processes.

Keywords

Component – Suitability criteria – Teaching practice – Didactic sequence and teacher training.

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* The authors take full responsibility for the translation of the text, including titles of books/articles and the quotations originally published in Portuguese.

1- Research, which has funding from the Vice-Rectory of Research and Postgraduate Studies of the University of Panama (UP) with the funds earned in the 2020-2021 code VIP-01-04-01-2021-04. It receives the support of the research Project FIED21-002 (SENACYT) entitled Competences and Knowledge of the Primary and Secondary Teacher for the Teaching of Mathematics in Hybrid Modality. It is gratefully acknowledged the collaboration of the research project PGC2018-098603-B-I00, funded by the Ministry of Science, Innovation and Universities, of the Government of Spain and PID2021-127104NB-I00, PID2019-104964GB-I00 (MICINN, FEDER, EU).

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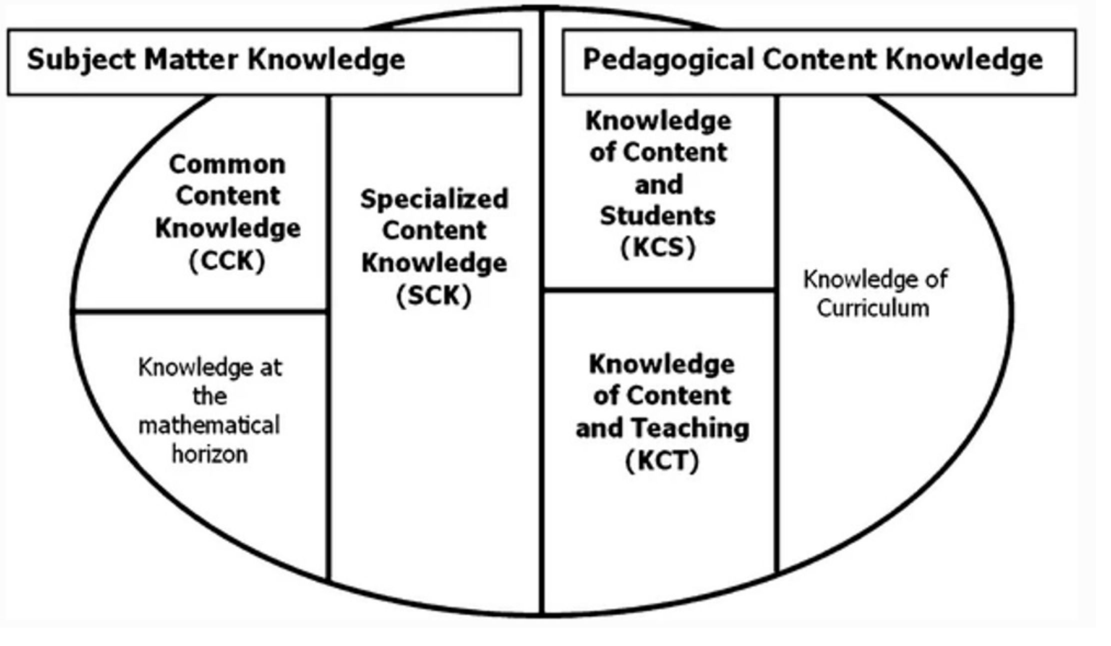
https://doi.org/10.1590/S1678-4634202349256706eng

Introduction

Mathematics teacher training is a widely studied and debated topic within Mathematics Education (SHULMAN, 1986; MASON, 2002; BISHOP et al., 2003; LESTER, 2007; HILL; BALL; SCHILLING, 2008; ENGLISH *et al.*, 2008). In this field of study, there are several theoretical approaches to the object of study. In this article, we take as a starting point the notion of mathematical knowledge for teaching (*Mathematical Knowledge for Teacher* - MKT), but we review it from the Onto-Semiotic Approach (OSA) (GODINO *et al.*, 2017). This notion emphasizes the study of mathematical knowledge that the teacher uses in his classes when teaching Mathematics to his students (HILL; BALL; SCHILLING, 2008).

According to the MKT approach, the professional knowledge of the mathematics teacher is made up of different knowledge, as shown in figure 1.

Figure 1- Mathematical Knowledge for Teaching (MKT)



Source: Hill, Ball, and Schilling (2008, p. 377).

According to Hill, Ball and Schilling (2008), the professional skills that a Mathematics teacher must have must allow him to carry out actions such as planning, implementation, explanation, assessment, and reflection on his teaching-learning processes. However, there is a difference regarding what tools teachers need to perform this type of specialized analysis, the knowledge necessary for such execution and how to evaluate processes related to teaching competencies. All these aspects would be part of what is called the teaching analysis competence within the knowledge and competencies of the Mathematics teacher (GODINO, GIACOMONE, BATANERO, FONT, 2017).

All this previous research has extensively studied the training of future teachers (or practicing teachers). However, fewer studies analyze or evaluate Mathematics teacher training programs (MORALES MAURE; GARCIA VAZQUEZ; DURAN GONZALEZ, 2019; GARCIA MARIMON *et al.*, 2021). Regarding the evaluation of teacher training in Mathematics, there are different international studies that focus on specific aspects of the professional teaching knowledge of the teacher of Mathematics, but do not give a more comprehensive and global vision. For example, the TEDS-M (International Comparative Study on Initial Mathematics Teacher Education) develops instruments to measure variables related to teacher competencies (TATTO *et al.*, 2008). Another study, the COACTIV project developed in Germany and used in the international PISA survey, shows a relationship between teachers' specialized knowledge and their students' learning (BAUMERT *et al.*, 2010). In South Korea, education authorities have successfully used the TET (*Teacher Education Test*) to assess mathematics teachers' necessary skills (KIM; HAN, 2002). In the case of the United States, the principles of evaluation were focused on improving aspects of student learning; however, in recent decades, there has been a shift in this approach to study how teachers can develop mathematics oriented to what is called high-quality (NCTM, 2000).

According to the assessment standards proposed by the National Council of Teachers of Mathematics (from now on referred to as NCTM), educators should be able to examine the effects of the mathematical tasks they propose, as well as the discourse and learning environment on students. They would also need to make instruction more responsive to the needs of students to ensure that each student is acquiring mathematical knowledge in an effective manner (NCTM, 2000).

This study analyzes the evaluation of the didactic training process of Panamanian teachers who participated in the research and emphasizes that instruction is an educator's activity, which must be able to master. Therefore, it is based on the premise that educators have to be able to reflect critically (and in a certain way also systematically) on their own educational praxis. For this, it is suggested that they take as a theoretical reference point the Didactic Mathematical Knowledge and Competences Model (DMKC) of the teacher, proposed by the OSA (GODINO; BATANERO; FONT, 2007, 2019; GODINO *et al.*, 2016; GODINO *et al.* 2017; FONT; BREDA; SALA, 2015; BREDA; SILVA; DE CARVALHO, 2016).

To evaluate educational practice, OSA uses what it calls "Didactic Suitability Criteria" (DSC), which is an analytical and evaluative construct that arises from a broad consensus in the area of Didactics of Mathematics (GODINO, 2013; BREDA; FONT; PINO-FAN, 2018).

Previous research on CIDs has shown that it is a consistent and methodologically robust construct to quantitatively analyze the didactic sequences developed by practicing mathematics teachers. Furthermore, these DSCs have been successfully used to evaluate the impact of a Diploma program aimed at the professional training of practicing teachers in Panama (GARCÍA MARIMÓN *et al.*, 2021). This research contributed to the development of DSCs as defined by Breda, Font and Pino-Fan (2018). The criteria proposed by these three authors were subdivided into components, which in turn, were disaggregated for a detailed understanding of the measurement process (evaluation) (GARCÍA MARIMÓN *et al*., 2021).

The purpose of this research is to expand the partial results found in this previous work (GARCÍA MARIMÓN *et al*., 2021) and to deepen the development of the components defined in the six Didactic Suitability Criteria (DSC) found in the didactic sequences carried out for the Program Didactic Strategies for the Teaching of Mathematics (DSTM). To this end, a qualitative analysis will be carried out on this occasion.[4]

Background to the study

In the study that was carried out at the beginning of the research project that frames the discussion proposed in this article (GARCÍA MARIMÓN *et al*., 2021), a quantitative measurement of the presence of the different components was carried out in which Breda, Font and Pino-Fan (2018) break down the six criteria that make up the DSCs (see Table 2). In this study, the data revealed that some of the participants in DSTM at the University of Panama had yet to consider several of the components corresponding to the six criteria of the DSCs. Table 1 shows the components that were little or not used by the program teachers.

Table 1- Assessment of components described in the DSCs to 10 participants of the previous research[5]

Component / Participant	Intra/interdisciplinary connections	Didactical Innovation	Autonomy	Representativeness of complexity	Errors	Time distribution	Social-professional practicality	Interests and Needs
Teacher A	0	0	0	1	3	3	3	3
Teacher B	0	0	3	2	3	3	0	3
Teacher C	0	3	0	2	3	3	0	0
Teacher D	0	0	3	2	2	3	0	3
Teacher E	0	3	3	2	3	3	0	3
Teacher F	3	0	3	3	1	3	3	3
Teacher G	0	3	0	0	3	3	0	3
Teacher H	3	3	1	2	3	1	0	3
Teacher I	0	0	0	2	0	0	0	3
Teacher J	0	0	0	0	3	3	0	0

Source: Authors.

4- Continuous Training Program of the University of Panama whose general objective is to provide training professionalizing supported by the OSA for all teachers who want to dedicate themselves to the teaching of Mathematics in primary education, which has a duration of four months (MAURE et al., 2021).

5- The values shown in Table 1 define the following: the value 0 indicates null presence of the component, the value 1 low presence of the component, the value 2 medium presence and the value 3 high presence of the component of the DSC.

The first row of Table 1 includes the eight components of the DSCs. The previous investigation revealed that the teachers participating in the Diploma program did not take this into account. We describe these components below (GODINO, 2011; BREDA *et al.*, 2018):

• Intra/interdisciplinary connections (a component of the ecological suitability that seeks that the topics developed are related with other mathematical contents or other disciplines).

• Didactical Innovation (a component of ecological criteria developed when educational research or reflection on teaching practice allows the introduction of different proposals in teachers' classes).

• Autonomy (a component of interactional suitability that shows when students assume responsibilities in the teaching-learning process).

• Representativeness of complexity (a component of the ecological suitability where the broad presentation in the curriculum of the mathematical notion to be taught is made).

• Errors (a component of the epistemic suitability that shows correct educational practices from the mathematical point of view).

• Time distribution (a component of the mediational suitability that reviews the moments of time adequacy to the needs found for the development of the proposed tasks).

• Social-professional practicality (a component of the ecological suitability related to the contents that transcend outside the classroom in issues of labor insertion).

• Interests and needs (a component of the affective suitability that is presented when intentionality that the concepts or tasks elaborated are of interest to the students).

Table 1 shows that many of the components described above were not developed by most teachers using the proposed quantitative evaluation system (GARCÍA MARIMÓN *et al.*, 2021). Because of this, the following research questions are posed: Why do mathematics teachers not use some components of the DSC in their plans? And, realizing that they use little (if anything) some of the components of DSCs, do those teachers modify their practice as a result of this reflection?

Theoretical framework

There are different theoretical approaches in the field of teacher education regarding the study of the mathematical and professional competencies of the teacher of Mathematics (SULLIVAN; WOOD, 2008; SILVERMAN; THOMPSON, 2008; STAHNKE; SCHUELER; ROESKEN-WINTER, 2016). In our study, we took the OSA approach. As explained above, the OSA works with the DMKC model, which coherently articulates the knowledge and skills necessary for teachers to teach Mathematics effectively (ideally). The constructs of the DMKC use the tools of the Onto-Semiotic Approach to Cognition and Mathematical Instruction (GODINO; BATANERO; FONT, 2007, 2019) both for the development of training devices (for example, didactic sequences, teaching units, etc.), as well as for their evaluation and critical reflection of the impact they have on students' mathematics learning.

According to the DMKC model, teachers must have two components, which are fundamental: mathematical competence and the competence of "analysis didactic and intervention" (RUBIO GOYCOCHEA, 2012). According to these authors, the competence of "analysis didactic and intervention" includes aspects such as the design, application and assessment of own learning sequences and others, using didactic analysis tools as quality criteria. The OSA has conducted various investigations on the DMKC model and its contribution to the training of mathematics teachers (FONT; BREDA; SALA, 2015; GIACOMONE; GODINO; BELTRAN-PELLICER, 2018; POCHULU; FONT; RODRIGUEZ, 2016; SECKEL, 2016).

DSCs are an analytical and evaluation construct created by Font and Godino (2011). These authors propose a construct composed of six criteria that analyze both the contents and the didactic competencies of mathematics teachers.

This OSA tool (BREDA *et al.*, 2018) serves to analyze the processes related to teaching practice according to the following six criteria:

- Epistemic suitability, which assesses whether the mathematics taught is good mathematics or not, or what is the same, the implication of the degree of representativeness of the concept proposed in the class in reference to its full meaning in the curriculum.
- Cognitive suitability, which evaluates the degree of learning achieved by students within an instruction process through stages of prior, intermediate and final evaluation.
- Ecological suitability, which serves to analyze the adaptation of the topics treated from the didactic activities developed in the class using the curriculum for their school and personal environment.
- Mediational suitability, which serves to evaluate how school media are adjusted as manipulative resources for the development of the didactic sequences presented.
- Affective suitability, which measures the degree of acceptance (motivation, interest, etc.) of the teachings received in the school system.
- Interactional suitability, which analyzes the interactions produced in the sequences presented in the classroom between students (interactions between equals), or students and teachers.

Font and his collaborators break down each of the criteria defined above into components, as shown in Table 2.

Table 2- Criteria and components with their descriptions

Suitability	Component	Component Description
Cognitive	Previous knowledge	Previous knowledge is necessary for the study of the subject.
	Adaptation of the curriculum to the individuals' different needs	Enlargement and reinforcement activities.
	Learning	Appropriation of the intended knowledge/skills.
	Cognitive demand	Relevant cognitive processes such as generalization, connections, changes in representation, etc.
Epistemic	Errors	They are correct educational practices from a mathematical point of view.
	Ambiguities	Conceptions can lead to confusion for students to understanding definitions, procedures or other aspects in school mathematics.
	Diversity of Processes	Relevant processes in mathematical activity such as modeling, argumentation, problem-solving, etc.
	Representativeness of complexity	Characteristic of the complexity contemplated in the curriculum of the mathematical notion to be taught.
Interactional	Teacher-student interaction	Communication developed between the teacher and the students for the tasks assigned in the teaching-learning process.
	Interaction amongst learners	Communication developed between students for the tasks assigned in the teaching-learning process.
	Autonomy	The student assumes responsibility within the teaching-learning process.
	Formative evaluation	Systematic observation of students' cognitive progress.
Affective	Attitudes	Promotion of involvement in activities, perseverance, responsibility, etc.
	Interests and needs	The concepts or tasks elaborated are of interest or needs of the students.
	Positive emotions	Promotion of self-esteem. Avoiding rejection, phobia or fear of mathematics.
Ecological	Adaptation to the curriculum	The contents, their implementation and evaluation correspond to the curricular guidelines.
	Intra/interdisciplinary connections	School content is related to other mathematical content or to content from other disciplines when developing different mathematical concepts.
	Social-professional Practicality	The contents they present transcend outside the classroom for issues of labor insertion.
	Didactical innovation	Introduction of new content, use of technological resources, and forms of evaluation, among other activities developed according to research or reflection of educational practices.
Mediational	Material resources	Use manipulative and computer materials to introduce the topics in the classroom.
	Number of students	The number and distribution of pupils make it possible to carry out the intended education.
	Timetable	Developed course schedule
	Classroom conditions	Adequacy of the classroom for the development of the intended instructional process.
	Time Distribution	Adequacy of time to the needs found for the development of tasks in the classroom

Source: Adapted by Font and other authors (2018).

The components with their respective criteria defined within the OSA serve for Mathematics teachers to reflect on their practice and assess their instructional processes. This paper focuses on discussing the reflection made by the teachers of Mathematics participating in the research when using the criteria of the DSC to analyze the design of their teaching practice (the units and didactic sequences), which include the activities that these teachers have developed, using the DMKC model with OSA tools. The contribution of this article is the qualitative discussion of the presence (or absence) of some of the components of the DSC (which was already detected in García Marimón et al., 2021), to analyze the subjective reasons adduced by these teachers to justify such absences (of certain components of the DSC, as summarized in Table 1).

Methodology

A qualitative research design is presented to answer the research question asked in this paper about the presence of defined components for DSC. This is a case study (STAKE, 1995): DSTM (experimental program based on the DMKC model) carried out at the University of Panama. The study subjects are described below, as well as the techniques for collecting the information and how the analysis has been carried out. We decided to conduct a qualitative study because we wanted to answer the two research questions presented. Beyond a quantitative description (which we already had in the previous study), in this study, we intend to understand the subjective reasons given by the participants in the previous study to justify their didactic decisions about using DSCs in their teaching planning. According to previous research, qualitative methodology is more appropriate to achieve this purpose.

Study subjects

The population studied are practicing teachers of the Panamanian educational system at the primary level. These teachers participated in the third DSTM training (class of 2019), whose program graduated 96 teachers from official and private schools in the country.

According to the guidelines suggested by Flick (2015) on how to select participants in a qualitative case study, teachers were selected who, in the previous study (GARCÍA MARIMÓN *et al.*, 2021) had stood out for the analysis carried out of their training activities using the components of the DSC. These teachers decided not to use some of the components of the DSCs (see Table 1), despite integrating their training as mathematics teachers. In an effort to answer our research questions, those teachers who in the previous research had not used (or had little used) the components of Table 1 were selected.

In this sense, a total of five teachers were intentionally selected from the third class according to the following selection criteria:

- Belong to the ten teachers studied in previous (GARCÍA MARIMÓN et al., 2021)
- Accessibility to be interviewed

- Varied representativeness in teaching experience (years of service)
- Varied weighting in the components is presented in Table 1.

All teachers selected for this interview participated in the previous quantitative research, knew in advance about the nature of the interview, and gave informed consent to participate.

Instrument for analysis

A semi-structured interview was designed to collect data from the answers to the two research questions addressed in this study. The semi-structured interview includes eight open-ended questions about each component shown in Table 1. A panel of experts validated this instrument in the Education and Mathematics Education area to avoid bias in the research. Each question aims to investigate the degree of presence of the components in the teaching practice of the teacher interviewed and, in particular, why some of these components have not had a more significant presence (or have not been taken into account) in the didactic sequences designed by the DSTM members who participated in the study. With this research instrument, data were collected from the five masters of the study, which allowed us to discuss why they had considered (or not) the different components of the DSC included in Table 1, based on the evidence collected in previous quantitative study (GARCÍA MARIMÓN *et al.*, 2021).

Table 3- First five questions of the semi-structured interview related to a component

Related component	Focused Interview Question
Intra/interdisciplinary connections	1. What are the obstacles or difficulties in making connections with other subjects or other topics of mathematics for the development of your class?
Intra/interdisciplinary connections	2. If you do not have difficulties, how often do you include this topic? Mention an example developed in your recorded class.
Didactical innovation	3. How do you implement innovation in the sequences developed in your class? Based on the understanding that innovation is when it is related to new content, technological resources, forms of evaluation or classroom organization
Didactical innovation	4. What self-assessment and self-improvement tools do you use in your classes?
Autonomy	5. How do you get the student to develop their own initiative for the construction of their learning (participate independently)? Describe a dynamic that allows for direct student participation

Source: Authors.

Table 3 shows some of the questions in the interviews conducted with teachers individually. It is a semi-structured interview so the questions were adjusted according to the willingness of the interviewees to answer each of them, to respect their natural speech.

About the analysis process

Once all the interviews were performed, the data collected during the fieldwork (the transcription of the interviews carried out with the selected participants of the Diploma), using discursive analysis (GEE, 2004), were examined. The transcription is organized into semantic units of meaning that were taken as a unit of observation to look for the relationship between the components and indicators of the DSCs developed within the analysis approach proposed by the OSA (BREDA *et al*., 2018).

According to Parker (2004), discursive analysis is designed as an alternative to a discursive action model that works by analyzing qualitative data through categories. In this case, the research work is an action model that relates the transcription fragments characterized with the codes agreed upon by the researchers (which correspond to the components and indicators described in Tables 1 and 2). For each of these identified components and indicators, a second analysis is performed to identify (measure) the degree of presence of that component or indicator, as shown in Table 4.

Table 4 – Description by semantic encodings of the presence of the connections component

Presence	Connections	Excerpts from coded transcription, at example level
3 = High	Intra/interdisciplinary connections have a high presence in the explanations given.	Well, when you want to do a study, you can do different topics, for example, your favorite subject, that is, what they handled. I went there along that line and wrote down all the subjects they gave me and the special subjects they gave to the special teachers. I said let's do a study on what is your favorite subject? Where everyone omits their opinion. From a natural conversation came the Statistics class.
2 = Average	He has an average presence of intra/interdisciplinary connections in the explanations given for the development of his classes	
1 =Low	Has a poor or low presence of intra/interdisciplinary connections in the explanations given for the development of their classes	Yes, you can achieve it sometimes, but we get stuck on one track.
0 = Null	Denies the use or presence of the component of intra/interdisciplinary connections in the explanations given for the development of his classes.	There are contents that I find difficult, and I have not been able to make that correlation because there are mathematical contents that do not lend themselves to correlate or develop them with another subject.

Source: Authors.

This qualitative analysis aims to identify the ideas shown by teachers according to the indicators previously presented and defined in this research. The methodological analysis is based on an interpretative process of the semantic units of meaning identified in the transcription of the study's interviewees. This analysis uses the Atlas ti software package (version 9.1.0) as a resource. Table 4 shows examples of the presence of the intra/interdisciplinary connections component with fragments extracted from the transcripts of the interviews conducted in the research.

Results of discursive analysis

Below are the results found in each of the questions asked in the interview with the five teachers who participated in the DSTM:

• In the first question, which is related to the component of inter/intradisciplinary connections of the ecological suitability of the DSC construct, we found that two interviewees indicated that there is no or little connection between Mathematics and other subjects. One respondent state, "[...] There is content that has been difficult for him, and I have not been able to make that correlation." Two indicate that it is very difficult to achieve these connections due to the lack of time to include this component in their learning sequences. However, most interviewees suggested that, according to their perception, the learning they achieve with their students is between medium and high when they include the inter/intradisciplinary connections component in their didactic proposal.

• In the second question, also related to inter/intradisciplinary connections, which inquiries about the frequency included in the designed didactic proposal, only two indicate that they would apply them in their classes. However, two indicated that including these types of connections (intra/interdisciplinary) can contribute to developing a high cognitive demand in their students.

• The third question investigates the didactical innovation component. The data collected shows that more than half of the people interviewed indicated that it is essential to use teaching resources, such as manipulative materials, to develop the didactical innovation component. One of the interviewees stated that he uses this component "[...] to work on the contents in a more concrete way and also reach the children depending on their different needs." Two considered that the didactical innovation component appears with a presence that they value as average (Table 4) in their teaching plans (proposed didactic resources).

• Regarding the fourth question, most people interviewed state that there is a high presence of developed learning, and they have positive emotional contact with their students throughout their didactic sequences.

• When inquiring about the autonomy component of the students, all the interviewees indicated that positive emotions always appear in their students, and most of them mentioned the presence of high interaction between the actors (teacher-student) within the teaching-learning process.

- Regarding the results related to epistemic suitability, specifically regarding the representativeness of the complexity of mathematical objects, most interviewees indicated that there is a need always to use manipulative or visual resources in their didactic sequences.
- In the seventh question, which refers to the possibility that the teachers who participated in the survey may make mistakes in the teaching-learning process they propose in their didactic sequences, three stated that they verify this process through a formative evaluation that helps them to make the necessary corrections. Some interviewees emphasize that there is the possibility of making mistakes when they do not make an adequate distribution of time in the didactic sequences because they want to achieve accelerated learning and do not pay attention to the accuracy or correctness of the content of the sequences they are designing. Only one argues that there must be an excellent adaptation of the curriculum by the teacher. In his response, he explains, "[...] We do a self-assessment and prepare ourselves 100% so there are no mistakes."
- The eighth question focuses on including the social-professional practicality component for the concepts developed in the didactic sequences. Four of the teachers indicated the need to adapt (medium to high, according to the scale of Table 4) of the curriculum for the effective achievement of the utility component in the topics studied. In addition, when including utility, three stated that they always observe positive emotions in students.

By way of summary, Table 5 shows the results described in the previous paragraphs. At the top of Table 5 is the presence coding for each component found in the semantic units of transcription. Each question has been placed in a row with its respective component, linking it with other components that appeared in the interviewees' answers. For example, in the case of the third question, the interviewees showed answers that took as reference components such as formative evaluation, representativeness of complexity, or material resources (Table 2) for the presentation of their ideas. In particular, the five teachers indicated always using (high presence) material resources when asked about the didactical innovation component of the ecological suitability criteria.

Table 5- Absolute frequencies were found among the components defined in the OSA for the interview

Component Presence Coding	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
Question 1 Connections	Learning				Time Distribution				Material Resources				Social-professional Practicality				Connections			
			1	2		2			1			1				2	1	2		
Question 2 Connections	Autonomy				Time Distribution				Representativeness of complexity				Previous knowledge				Connections			
				1		1					1			1						2
Question 3 Didactical Innovation	Formative evaluation				Representativeness of complexity				Diversity of Processes				Material resources				Didactical Innovation			
				2			2			1						5				2
Question 4 Didactical Innovation	Positive Emotions				Learning				Formative Evaluation				Adaptation of the curriculum to the individuals' different needs				Didactical Innovation			
				2				3				1			1			1	1	
Question 5 Autonomy	Positive Emotions				Attitudes				Teacher-Student Interaction				Previous knowledge				Autonomy			
				4				2				3			1					2
Question 6 Representativeness of complexity	Adaptation to the Curriculum				Diversity of Processes				Material Resources				Social-professional Practicality				Representativity of complexity			
				1			2				2	2			1	2		1	1	
Question 7 Errors	Time Distribution				Formative Evaluation				Learning				Previous knowledge				Errors			
	2							3		1		1			1				3	
Question 8 Social-professional practicality	Positive Emotions				Adaptation to the Curriculum				Teacher-Student Interaction				Interests and Needs				Social-professional practicality			
				3			1	3				1		1		1				3

Source: Eown work.

Discussion

The necessary mathematical competencies of a teacher can be assessed using the CID (BREDA; FONT; PINO-FAN, 2018; BREDA *et al.*, 2018; FONT; BREDA; SALA, 2015; FONT *et al.*, 2018). In this paper, we try to analyze what happens with the following components of this construct: connections, autonomy, social-professional practicality, errors, representativeness of complexity, and educational innovation, which according to a previous study (GARCÍA MARIMÓN *et al.*, 2021) seem not to be used, or are little used by teachers in their teaching plans.

The NCTM (2000) argues that to ensure that students develop their advanced mathematical thinking, they must be able to carry out a series of mathematical processes, such as the use of *connections* in the contents studied. However, the teachers who participated in the study by García Marimón and other authors (2021) mostly state that they encounter complications when they try to use the component of inter/intradisciplinary connections. When asked, they don't delve into why they don't use this component or why it isn't very easy to do so. According to Morales Maure (2019), teachers in Panama have limited academic mathematical training. This translates into a limitation as regards their degree of competence over the specialist knowledge of the subject of Mathematics – one of the key aspects highlighted in approaches such as MKT (HILL; BALL; SCHILLING, 2008) in the professional training of the teacher of Mathematics-. This limitation in the specialized knowledge of Mathematics, which we have been able to verify in the study discussed in this article, could also mean that teachers who have participated in the study have a low teacher of mathematical concepts. This aspect also makes it difficult for them to integrate the component of connections (both inter/intradisciplinary) in their teaching programs.

On the other hand, the interviewees relate the didactical innovation component with having manipulative resources for the development of their classes. According to Chisag and other authors (2017), the use of diverse interactive didactic resources motivates students, promoting the development of their mathematical thinking so that didactical innovation contributes to improving the teaching and learning process in the classroom. The teachers who have participated in this study agree with this statement. The results are consistent. They think that teaching innovation (in particular, using non-traditional resources in the classroom) enhances learning, motivates students more, and helps them develop a deeper understanding of those mathematical concepts.

When examining the autonomy component, the data indicate that the teachers participating in the study agree in suggesting that the existence of interactions between students/teachers, or between students/students, has a positive impact on the attitudes that students develop around the subject of Mathematics. This result is also consistent with previous research in communication analysis within the classroom (FARSANI; BREDA; SALA, 2020). In fact, Meduca (2012) indicates that a fundamental piece of the Panamanian curriculum is to develop the competence of autonomy and personal initiative. Hence, teachers are committed to developing it; in this case, the teachers interviewed always perceive a positive climate in the school environment around this component.

First, however, it needs to be clarified what strategies they use to promote such autonomy. On the other hand, Ramírez Esperón and other authors (2019) also state that promoting autonomy among students has an essential effect on teaching and learning since they have more facilities (greater opportunities) to be able to respond positively when the teacher uses two or more different didactic procedures or strategies (diverse teaching) when developing a specific topic. For this reason, these authors affirm that most teachers maintain that autonomy is critical to promoting classroom communication and allows students to internalize different didactic strategies. The evidence collected and discussed in this study is consistent with this result and verifies it in the case of the DSTM object of this study.

The next component discussed here is the one that refers to resources. Material resources are not necessarily the solution to learning problems, but they introduce different ways of representing a mathematical concept in the classroom, facilitating its understanding (RIVEROS; MENDOZA BERNÁL; CASTRO, 2011). Indeed, most of the teachers interviewed agree that when they use teaching resources, they offer various forms of representation of the concepts (mathematical) they wish to teach. Most of the scientific literature in mathematics education is consistent with this result (BARTOLINI; MARTIGNONE, 2020; BREDA; DO ROSARIO, 2016; JIMENEZ; ESPINOSA, 2019).

Regarding the component of errors, Di Blasi Regner and other authors (2003) group many difficulties associated with generating errors, as is the particular case of the difficulties related to teaching processes. That is, the school institution must promote processes in the classroom that reduce learning difficulties. In this case, some teachers participating in this study indicated that the short time they had available to give the lesson motivated some of them to make mistakes in their explanations. Faced with this, some of them mentioned that reviewing previous knowledge is essential to avoid making mistakes during class. However, only one of the teachers mentioned that having solid preparation (Mathematics) is essential to prevent this type of situation. This result is consistent with what other authors say, such as Brousseau, David, and Werner (1986). According to Rico (1995), the error results from inadequate conceptions of the concept, which, in this case, happens to some of the participating teachers.

Finally, the data also suggest that the development of positive emotions and curricular adaptation are components that appear when interviewees talk about the component of social-professional practicality in the development of schoolwork. Indeed, Albiol (2020) mentions that teachers indicate that most of the examples, activities, and problems are contextualized (curricular adaptation) and are related to real-life situations, such as the student's environment (utility). This indicates that our interviewees understand the close relationship between utility components and curricular adaptation.

Final considerations

The objective of this study has been to analyze why there are some components of the DSC that, despite integrating teacher training, are not used (or are not given importance) in the plans prepared by some of the teachers of the Diploma program object of study and, if by the fact of participating in this qualitative study, these teachers modify in some way their current aulic.

This work initially allows teachers to reflect on their educational practices, which will enable them to make the necessary modifications to incorporate elements that have been developed from mathematical educational research and that have yielded positive fruits to improve teaching processes with the DSC (GODINO, 2013). Teachers recognize the importance of these components (BREDA *et al*., 2018), as is the case of connections and didactical innovation or when they indicate the high presence of learning in their students.

From the review of the data found by García Marimón and other authors (2021), we realized that DSTM teachers should have used several of the components and indicators of the DSC (Table 1). This finding generated we wonder what was occurring since, as Breda and collaborators (2018) explain, the CIDs arise from a broad consensus in which teachers from various countries around the world have participated (BREDA; FONT; PINO-FAN, 2018), so it would be expected that this consensus in the use of these components and indicators would also be given in the EDEM program. However, this was not the case (GARCÍA MARIMÓN *et al*., 2021). The results obtained after the qualitative work performed through the interviews we conducted suggest that all the teachers participating in the study and the previous study did not use all the components and indicators included in Table 1. However, directly or indirectly, they stated in their responses that they agree that these components defined for DSCs should be part of the development of their didactic sequences. Therefore, when the voices of these teachers are included in the design and process of the research itself, we conclude that they realize how they are using the DSCs and the process of participation in the research itself causes them to change their initial positioning and modify their practices. In the end, in all cases, it was agreed that teachers of the subject of Mathematics must have at their disposal theoretical constructs such as the DSC because using them critically allows them to improve their teaching practice, as well as their competence in "analysis didactic and intervention" (GODINO; FONT; BATANERO, 2020).

Of course, this conclusion cannot be generalized or extended to all Mathematics teachers since the analysis was carried out with a sample of the previous case study published by García Marimón and other authors (2021). The previous study includes the entire group of teachers who participated in the DSTM program, the only one that exists in Panama with the development of reflection in their Mathematics teaching practice. To carry out this work, a small sample (five cases) was selected, from which they showed the most discordant results in the previous study. The motivated selection of the sample does not allow the results to be generalized because the aim was to understand why they had not used the components and indicators of Table 1. However, based on the answers of these five teachers, we have been able to verify the coherence of some of their answers with previous results of research in the field of mathematics education, and the need to continue investigating to confirm whether the rest of the answers can be (or not) generalized for teachers as a whole. Of course, questions remain open, such as: Why can›t teachers connect to the contents covered in a class? What are the limitations that arise for the implementation of this component or other components in its sequences? To answer these questions, future research will be needed.

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Received on September 29, 2021
Reviewed on March 02, 2022
Approved on April 22, 2022

Editor: Prof. Dr. Marcos Sidnei Pagotto-Euzebio

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