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An Analysis of Federal Entities' Compliance with Public Spending: Applying the Newcomb-Benford Law to the 1st and 2nd Digits of Spending in Two Brazilian States*

José Isidio de Freitas Costa
Master's from the Graduate Program in Accounting, Federal University of Pernambuco
E-mail: jisidio@tce.pe.gov.br

Josenildo dos Santos
Associate Professor, Department of Accounting and Actuarial Science, Federal University of Pernambuco
E-mail: jsnipcontabeis@yahoo.com.br

Silvana Karina de Melo Travassos
Master's from the Graduate Program in Accounting, Federal University of Pernambuco
E-mail: silvanakmt@yahoo.com.br

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ABSTRACT

The following question is investigated in this study: Are there significant deviations in the distribution of the 1st and 2nd digits of state public spending from the behavior predicted by the Newcomb-Benford Law (NB-Law)? The goal of this study is to detect the occurrence of significant deviations in the distribution of the 1st and 2nd digits of state public spending compared to the standard distribution defined in the NB-Law. This study develops an interdisciplinary and exploratory methodology to analyze 134,281 contracts issued by 20 management units in two states. An accounting model based on hypothesis testing was applied to analyze the data, thereby evaluating the conformity between the observed distribution and that predicated by NB-Law. This study showed that there were significant differences in the distribution of digits; the numbers 7 and 8 had excess occurrences, and 9 and 6 were rare occurrences compared to the proportions expected by the NB-Law for the 1st figure. This behavior denoted a tendency to avoid conducting the bidding process. The analysis of the 2nd digit, unprecedented for the Brazilian case, showed a significant excess of occurrences for the numbers 0 and 5, which indicated the use of rounding in determining the value of contracts. We show the feasibility, usefulness, and practicality of applying the NB-Law to the action of oversight bodies, particularly when planning an audit and determining the audited sample.

Keywords: Public Spending. Newcomb-Benford Law. Second digit. Continuous auditing.

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1 INTRODUCTION

Without the rigor of establishing precise data, it is evident that accounting has used various resources and techniques to complete the basic aim of providing economic information to users to enable rational decisions (Iudícibus, 2009). Mankind's effort in searching for asset control can be observed from the use of stones and wooden objects in prehistory, as shown in the pictorial records in caves (primitive drawings), to the current use of information systems that process data in a computer network capable of global connections.

The evolution of information technology, particularly the increasing ease of storing and accessing large volumes of data, created fast-paced changes in the techniques and processes used in various areas of knowledge, particularly accounting. An example of these changes was the shift in the methods with which business transactions are initiated, recorded, processed, and reported with great effects on the form and content not only in the corresponding accounting records but also in the nature of audit evidence (Nearon, 2005).

Documents and accounting records on paper have been systematically replaced by electronic transactions. Mandatory accounting books, including the Journal Entries and General Leadger, were often exclusively handwritten and followed the intrinsic formalities of bookkeeping, such as the chronological sequencing of records and absence of erasures and interlineations. Currently, because of the use of computers and accounting systems that integrate records and budgetary, financial, and property controls from the three spheres of government, the Public Administration has a large volume of information stored in computerized databases. The structuring of information in electronic databases has significantly contributed to its analysis, both for the possibility of organizing and correlating data and the speed of data processing.

Moreover, conducting transactions electronically has enabled the development of concurrent control, which is executed in an environment of continuous auditing. Concurrent control is characterized by its promptness and greater effectiveness because it is exercised during or shortly after the occurrence of the events it controls. The proximity of this action not only provides auditing with a greater chance of success but also produces the possibility of restraining the effects of an irregular and perhaps undetected act.

In this context, which expands the limitations of time and volume in processing information, a new range of possibilities arise from timely interferences in large populations or samples. Among the possible applications, the aim of this study is to detect the occurrence of significant deviations in the distribution of the 1st and 2nd digits of state public spending compared to the standard distribution from the Newcomb-Benford Law (NB-Law).

The NB-Law consists of an anomaly of probabilities and shows that smaller numbers occur with greater frequency compared to larger numbers in the 1st digit position. Therefore, lower numbers have a higher probability of occurrence than higher numbers, so the numbers 1, 2, and 3 are more frequent than 4, 5, 6...9.

The use of the NB-Law as a methodology applied to public sector auditing was introduced in Brazil by the authors Santos, Diniz, and Ribeiro Filho (2003); Santos and Diniz (2004); and Santos, Diniz, and Corrar (2005). Occasionally, an accounting model based on the relationship between the NB-Law and hypothesis testing introduced by Carslaw (1988) is used (Z-test and $\chi 2$ test) to identify deviations in the distribution of the 1st digit extracted from values in expenditure notes of municipalities in Paraiba.

The following question is investigated in this study: Are there significant deviations in the distribution of the 1st and 2nd digits in state public spending compared to the behavior predicted by the NB-Law?

There is a consensus that the binomial valuation of controls and sample analysis of data have been the basis of private auditing, whose methodology is accepted and applied globally. Through the use of quantitative methods applied to public sector auditing, the proposal presented in this paper identifies nonconformities in the behavior of public spending compared to the proportion of occurrences predicted by the NB-Law for digits in the 1st and 2nd positions. Similar applications were observed in the national and international literature, associating the occurrence of deviations in the behavior of digits to the possibility of repeated error and fraud.

Berton (1995) developed a computer program with an application to detect financial fraud using Benford's Law. Bhattacharya (2002) noted that some investigative models can assist auditors in tracing the perpetrators of financial fraud, thus showing that the set of tools available to the forensic accountant may be substantially enhanced by combining the fraud classification system with the mathematical elegance of Benford's Law. Das and Zhang (2002) reached the identical conclusion as Carslaw (1988) and Caneghem (2002) regarding the manipulation of earnings related to profit shares in communication companies through observing the practice of rounding up in transactions to positively affect corporate earnings and performance.

Nigrini (2005) identified the problem of the American company Enron in 2001, noting that the sequence of events that culminated in the company's bankruptcy filing was triggered by changing the data in financial statements. In that study, the author used Benford's Law to evaluate whether there was a detectable change in the data included in the earnings reports during the fiscal years of 2001 and 2002. The author concluded that there was an increase in the revenue amounts reported by the company's management.

Geyer and Williamson (2004) emphasized the necessity for governments, for fiscal purposes, and corporations in their internal controls to detect fraudulent patterns in their financial data. The authors discussed the method of statistical detection developed by Nigrini to test the compliance of a dataset with the NB- Law. Moore and Benjamin (2004) presented a case study using the application of digital analysis combined with Benford's Law for costs in a small chemical production plant. Their analyses detected suspicious purchasing operations, which resulted in the discovery and reporting of fraudulent activities. Therefore, the indications of deviations observed in the analysis were transformed into an aid for the formation of a sample to be audited.

From a managerial standpoint, the deviations observed in analyzing purchasing behavior can be formulated into a scale among the audited entities (jurisdictions). This scale will act as a gauge of management and lend itself as another synthetic index to support managerial decision making with repercussions for optimizing the results by allocating resources to the prioritized areas or entities. This research is also justified because it is the first study to use the NB-Law as the basis of an interdisciplinary methodology for an applied analysis of public spending in Brazilian states and it introduces Brazil to the application of the NB-Law for the 2nd digit.

This study consists of five sections and begins with this introduction. The following section presents a theoretical background on this topic, the third section discusses the methodology used in this research, the fourth section presents the analysis and interpretation of the results, and the fifth section presents the concluding remarks.

2 THEORETICAL BACKGROUND

2.1 Continuous Auditing.

There are several definitions of auditing, including definitions related to financial and non-financial assets, which shows that auditing is a branch of accounting that can be applied as a useful tool to other sciences. Auditing consists of performing a systematic and official review to ensure that the applicable system, program, product, service, and process cover all of the required characteristics, criteria, and parameters. Namely, auditing is a mechanism that aims to control and monitor processes and standards, whether for a public or private company or sector (Attie, 2010).

Therefore, auditing can be defined as the collection, study, and systematic evaluation of transactions, procedures, routines, and financial statements of an entity with the goal of providing the users with an impartial opinion on their compliance based on norms and principles (Perez Júnior, 1998).

Motta Júnior (2010) noted that various technological advances have caused rapid and profound changes in organizations and society. Consequently, auditing techniques are undergoing changes to adapt to the demands of this new scenario with the emergence of continuous auditing as a direct reflection of these changes.

According to Vasarhelyi and Halper (1991), continuous auditing is characterized by producing results simultaneously with or shortly after the occurrence of a relevant controlled event. Therefore, the continuous auditing process depends on the presence of a computerized control system and electronically stored data.

The best attribute in the exercise of concurrent control in the continuous auditing process compared to the traditional model of a posteriori control is timing because it performs its functions in parallel with the occurrence of the controlled events. Based on the speed of its implementation, which contributes to the possibility of restraining the effects of a detected potentially irregular act, concurrent control is more efficient. However, Murcia, Souza and Borba (2008) noted that the cost of concurrent control implementation would only be economically viable with an automated execution.

Although the deployment of automated auditing routines is a reality, particularly in the area of risk analysis in lending administered by credit card companies, its use remains promising compared to its potential. An example of this is its application to auditing in the public sector: the performance of automatic tests to verify compliance with spending limits before values of expenditures; verify the tax compliance of the providers of goods and services to the administration; and monitor deviations in spending behaviors compared to the projected value. In this study, we emphasize the results of a practical application, in which accounting methods were used to analyze the behaviors of the 1st and 2nd digits in allocation bills and detect respective deviations compared to the distribution predicted by the NB-Law based on an interdisciplinary methodology used by Carslaw in 1988.

2.2 Bidding Limits for Public Spending.

Article 37, paragraph XXI, of the Federal Constitution established that except in cases provided by law, the Public Administration's works, services, purchases, and disposals shall be contracted through a public bidding process, ensuring equal conditions to all competitors regarding the constitutional principal of isonomy.

It became the responsibility of Federal Law n. 8,666/93, the Law of Bidding and Administrative Contracts, to establish the standards regarding the bidding process, which is intended to select the most advantageous proposal for the administration. The process performs this function through certifying the requirements related to the technical and economic-financial capacity of the bidders and the quality and fair value of the auctioned object. Therefore, bidding modalities and value limits were established for the public sector prior to contracting purchases, works, and engineering services, as shown in Table 1.

Table 1 *A summary of modalities and bidding limits*

Modality	Purchases – R$	Works and Services – R$
Exemption	Below 8,000	Below 15,000
Invitation	Between 8,000 and 80,000	Between 15,000 and 150,000
Price Taking	Between 80,000 and 650,000	Between 150,000 and 1,500,000
Competition	Above 650,000	Above 1,500,000
Proclamation	Applies to any value	Applies to any value

Source: Adapted from Federal Laws n. 8,666/93 and 10,520/02.

One of the irregularities studied in the processing of public spending by external control agencies is the reduction of public spending to values lower than or equal to the spending limit, which is a practice used to circumvent the bidding process and direct public spending to the desired vendor.

2.3 The NB-Law.

The NB-Law, which was empirically discovered by the mathematician and astronomer Simon Newcomb in 1881 and apparently verified independently by the physicist Frank Benford in 1938, constitutes an anomaly of probabilities and shows that smaller numbers occur more frequently in the 1^{st} digit compared to higher numbers. In a reasonably sized sample of random numbers obtained from a data source, common sense compels us to believe that the 1^{st} significant digit (excluding zero) could be any number between 1 and 9 and that these values would be equally probable. In this situation, the NB-Law shows that the smaller numbers 1, 2, and 3 have a greater probability of occurrence, approximately 60.2%, compared to the other numbers 4, 5, 6...9. This anomaly was also verified with less intensity regarding the distribution of the 2^{nd} digit.

2.3.1 The Probabilities of the NB-Law for the 1^{st} and 2^{nd} Digits.

Newcomb (1881) initially calculated the probability of occurrence for the First Significant Digit (FSD) bases on his empirical findings. Hill (1996) showed the mathematical expression of laws governing the 1^{st} (significant) and 2^{nd} digits of random distributions of numbers observed by Newcomb.

Prob (1^{st} significant digit = d): **1**

$$log_{10}\,(1+d^{-1})$$

Prob (2^{nd} digit = d): **2**

$$\sum_{k=1}^{9} \log_{10}\,(1+(10k+d)^{-1})$$

Prob (d) = Probability of occurrence of digit d in any number;

(1) d = 1^{st} significant digit belonging to the set of whole numbers between 1 and 9; and

(2) d = 2^{nd} digit belonging to the set of numbers (digits) between 0 and 9.

With the above equations, the probability of occurrence for the 1^{st} and 2^{nd} digits can be expressed in Tables 2 and 3, respectively.

Table 2 *The probability of occurrence of the 1^{st} significant digit*

Digit (d)	0	1	2	3	4	5	6	7	8	9	Total
P(d)	-	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046	100.0

Source: Adapted from Carslaw (1988).

Table 3 *The probability of occurrence of the 2^{nd} digit*

Digit (d)	0	1	2	3	4	5	6	7	8	9	Total
P(d)	0.1197	0.1138	0.1088	0.1043	0.1003	0.0967	0.0934	0.0904	0.0876	0.085	100.0

Source: Adapted from Carslaw (1988).

2.3.2 *Deductive Analysis for the Occurrence of this Phenomenon.*

Newcomb (1881) and Benford (1938) stated that in nature, there are more numbers beginning with smaller digits than larger digits. This claim is aligned with the factual limitation resulting from a scarcity of resources. In turn, this can be better understood in the following claims: Considering the limitation of food resources, is there a higher occurrence of large or small living organisms? Considering the limitations of economic resources, is there a higher incidence of large multinationals than small and medium enterprises? Considering the limitations of financial resources, is there a greater occurrence of costs with large or small monetary values?

Nigrini (1999) provided an intuitive explanation of the NB-Law to show how it functions in a scenario of exponential growth using, for example, the total assets of a mutual fund that is growing at a rate of 10% per year. When the total assets are R$100 million, the 1st digit of the total assets is 1. The 1st digit will continue to be 1 until the total assets reach R$200 million. This will require an increase of 100% (100 to 200); however, for the above proposed growth rate of 10% per year, it would take 10 years to reach R$200 million. With R$500 million, the 1st digit is 5. Growing at a rate of 10% per year, the total assets would require 2 years to increase from R$500 million to R$600 million, signifying less time than for the assets to grow from R$100 million to R$200 million. With R$900 million, the 1st digit will be 9 until the total assets reach R$1 billion. Increasing at a rate of 10% per year, the total assets would require 1 year and 1 month. The above reasoning applies analogously to a constant rate of population growth or decline.

Studies in the field of Probability Theory, including Hill (1995, 1996), Pinkham (1961) and Raimi (1969), show that the NB-Law applies to datasets with the following properties: (a) those that are scale invariant and (b) those that derive from a selection process using a variety of different sources. This result is obtained from a more rigorous analysis of the Central Limit Theorem in the form of theorems for the mantissa of random variables on the effect of multiplication. Accordingly, when the number of variables increases, the density function tends towards a logarithmic distribution. Hill (1996) rigorously showed that "the distribution of the distribution of random numbers" obtained from random samples that arise from a variety of different distributions is a Newcomb-Benford distribution.

2.4 Cognitive Reference Points.

Psychological studies conducted by Moyer and Landauer (1967) showed that humans, in their cognitive process of perception and comparative judgment of numerical magnitudes, use a "symbolic effect of distance". When individuals were shown numerical pairs to identify the element of greatest value, these studies showed that the decisions were faster when the difference between the numbers was large, for example, 3 and 9, a large difference, produced a faster response and 3 and 4, a small difference, produced a slower response. In studying the response time when identifying the magnitude for pairs of equidistant numbers, Banks et al. (1976) showed that the human perception of the difference between numbers follows a psychophysical function; determining the distance between pairs of numbers evokes a faster response when the absolute values are smaller. For example, 1 and 2 (low pair and faster response) and 7 and 8 (high pair and slower response) suggest the occurrence of a logarithmic function of understanding (logarithmic counting scale) or a similar function associated with numerical magnitude. The results from these studies indicate that the concept of magnitude is more easily identified in smaller numbers (1, 2, and 3) compared to larger numbers (7, 8, and 9); therefore, smaller numbers function as reference points in determining comparative decisions of the magnitude between numbers.

The use of reference points in the comparative process of perception and judgment of magnitudes for numbers composed of several digits with a focus on the decimal system (0, 1...9) was also identified by Gabor and Granger (1996). The authors showed in their study that humans use numbers that are multiples of ten as a reference to evaluate the size of other numbers. In this comparative process, these key numbers function as benchmarks that will be compared to distances relative to other numbers (Rosch, 1975). The emphasis given to numbers that are factors of ten promotes a tendency to round up or down when a person observes a number. For example, when the number 3,979 or 4,012 is observed, there is a tendency to evaluate its magnitude as 4,000.

Carslaw (1988) noted the occurrence of this phenomenon and analyzed the frequency distribution of the numbers 0 to 9 for the 2nd digit of revenues and net income in the financial statements of 220 companies in New Zealand. By comparing the observed distribution to the frequencies predicted in the NB-Law for the 2nd digit, the author observed that the number (0) showed a deviation of excess occurrence, which was interpreted as evidence of rounding up in values published by companies. For the author, the pressure for managers to reach company goals would promote a tendency of rounding up whenever there was uncertainty related to the event to be recorded.

3 METHODOLOGICAL PROCEDURE

Regarding the scientific approach, this study is the product of an interdisciplinary study strategically based on a qualitative/quantitative/qualitative analysis. The pillars are accounting and mathematical sciences (probability theory, statistics, and computer science) that are applied to the continuous auditing of public accounts. Raupp and Beuren (2008) explain that quantitative research uses statistical instruments to analyze data. According to Denzim and Lincoln (1994), qualitative analysis corresponds to a set of operations required for the systematization and formation of a coherent process of collecting, storing, and retrieving data. Martins and Theóphilo (2009) noted that it is unreasonable to believe there can be an exclusively qualitative or quantitative study. Scientific research includes both.

This research can be characterized as an exploratory study because it attempts to discover data and information for a better understanding of the phenomenon in question; namely, the occurrence of significant deviations from the NB-Law in the distribution of the 1st and 2nd digits for state public spending. An exploratory study can be defined as one of the main forms of knowledge construction, which incorporates unique characteristics and allows the researcher to increase his/her experience regarding a particular issue (Raupp & Beuren, 2008; Triviños, 1987).

The present study investigated two states, which were chosen based on data accessibility. The data could be obtained from their respective official sites available on the Internet. The two states are in the northeastern region of Brazil. For isonomic treatment, we chose not to identify the names of the states because the study was conducted in only two of the states in the Northeast. Accordingly, identifying the names could be interpreted as an unfair penalty relative to other states that were not affected by the analysis. The sample size was selected deterministically and comprised of 20 management units (Unidades Gestoras – UGs), 10 for each state, from among the units with the highest spending volume and a minimum of 800 contracts issued.

The selected sample represented 44.51% of the total spending for the analyzed states. After choosing the sample of UGs, all contracts issued by the units for the 2009 fiscal year that were equal to or greater than R$1.00 were obtained. This ensured that all the observations, 134,281 altogether, had a 1st non-zero digit. From observations of the values of each contract, the 1st and 2nd digits were then extracted and grouped separately in the identical level of observations with Microsoft Excel version 2007.

The accounting model introduced by Carslaw (1988) was applied to the data analysis to address the proposed question and meet the general goal of this study. This model is used in hypothesis testing (Z-test and χ2 test) to evaluate the agreement in the distribution of a dataset (Po) compared to the percentages predicted by the NB-Law (Pe). For Nigrini and Miller (2006), the detection of these deviations could be applied with a second-order test related to Benford's Law and is used by auditors to detect fraud, errors, or simulated data.

A Z-test was used as a measure of statistical significance in determining the differences between the observed (Po) and expected (Pe) probability distributions. The tests were applied separately for each digit, using the following formula:

$$Z=\frac{|Po\text{-}Pe| - \frac{1}{2n}}{\sqrt{\frac{Pe(1\text{-}Pe)}{n}}}$$

where n is the number of observations and 1/2n is the continuity correction term and is only used when it is less than |Po - Pe|. A significance level of α = 0.05 was adopted with a $Z_{critical}$ value equal to 1.96.

Based on the research question in this study, "Are there significant deviations in the distribution of the 1st and 2nd digits in state public spending compared to the behavior predicted by the Newcomb-Benford Law?", the following hypotheses were established for the analysis of deviations for each digit:

H_{0A}: *Po=Pe* - There is no statistically significant difference for the 1st digit;

H_{1A}: *Po≠Pe* - There is a statistically significant difference for the 1st digit;

H_{0B}: *Po=Pe* - There is no statistically significant difference for the 2nd digit;

H_{1B}: *Po≠Pe* - There is a statistically significant difference for the 2nd digit.

The χ2 test was used to determine whether the two probability distributions were consistent with one another overall. This test verified whether the observed frequencies of the digits as a whole followed the distribution predicted by the NB-Law by applying the following formulas to the 1st and 2nd digits:

$$\chi^2=\sum_{d=1}^{9}\frac{(PO\text{-}PE)^2}{PE} \quad \text{and } \chi^2=\sum_{d=0}^{9}\frac{(PO\text{-}PE)^2}{PE}$$

where PO and PE are the observed and expected proportions defined by PO = (Po) x (no in population) and PE = (Pe) x (no in population). A significance level of α = 0.05 was adopted with 8 (1st digit) and 9 (2nd digit) degrees of freedom, and critical values for χ2 equal to 15.507 and 16.919 were obtained.

Based on the research question presented in this study, "Are there significant deviations in the distribution of the 1st and 2nd digits in state public spending compared to the behavior predicted by the Newcomb-Benford Law?", the following hypotheses were established to analyze the deviation of the sequence of all digits:

H_{0C}: *PO=PE* - There is no statistically significant difference for the 1st digit;

H_{1C}: *PO≠PE* - There is a statistically significant difference for the 1st digit;

H_{0D}: *PO=PE* - There is no statistically significant difference for the 2nd digit;

H_{1D}: *PO≠PE* - There is a statistically significant difference for the 2nd digit.

One noteworthy limitation in this study is that fieldwork was not conducted. The analyses and conclusions reflect the behavior of data on public spending obtained from official websites.

4 ANALYSIS OF THE RESULTS

Table 4 presents the results of the Z-test and χ2 test applied to the individual analysis of the 1st digit for the 10 UGs in the sample from state E1. The results observed for each UG show that the values of the Z-statistic were below the $Z_{critical}$ for the established level of significance in 25 tests, thus accepting the null hypothesis H_{0A} for such occurrences. In the 65 remaining tests, the results of the Z-statistic were above the $Z_{critical}$, which led to the acceptance of the alternative hypothesis H_{1A} for these digits. All UGs showed at least 4 numbers with significant deviations, from UG8 with deviations in 1, 3, 8, and 9 to UG3 with deviations in all numbers. Table 4 shows the results of the χ2 test applied to the overall analysis of deviations in all numbers. These results reject the null hypothesis H_{0C} for all UGs, from UG8 with the smallest deviation (44.47) to UG3 with the largest observed deviation (3,072.91).

Table 4 *The individual analysis of each management unit in state E1 applied to the 1st digit - Z-test and χ^2 test*

	Digit	UG1	UG2	UG3	UG4	UG5	UG6	UG7	UG8	UG9	UG10
Z-Test	1	1.789*	11.460	24.177	0.408*	13.431	8.269	1.965	3.059	6.064	8.463
	2	3.180	28.627	15.450	2.532	4.308	0.578*	0.204*	0.077*	7.514	3.054
	3	2.660	8.476	8.994	0.915*	4.793	2.809	3.541	2.451	3.329	5.539
	4	6.122	19.983	4.021	7.983	9.156	0.237*	2.040	0.753*	4.582	0.712*
	5	2.841	3.852	11.690	3.296	2.011	8.450	1.686*	0.686*	1.084*	1.642*
	6	2.152	17.603	2.158	1.457*	19.453	14.311	5.394	1.859*	2.534	1.323*
	7	0.002*	0.399*	44.579	0.660*	8.790	8.704	3.848	0.888*	2.534	4.734
	8	3.608	1.342 *	14.123	3.364	0.690*	8.529	0.305*	3.940	0.837*	4.929
	9	4.641	13.885	14.029	2.301	1.451*	2.524	2.293	3.526	3.778	4.296
χ^2-Test		97.28	1,682.44	3,072.91	93.63	672.57	460.71	67.42	44.47	131.02	152.73

(*) No significant difference at α = 0.05 with $Z_{critical}$ <= 1.96 and $\chi^2_{critical}$ <= 15.507.

Drawing on the above-mentioned methodology, Table 5 shows the results of the Z-test and χ2 test for the 1st digit, which were applied individually to the 10 UGs in the sample from state E2. It can be observed in Table 5 that the Z-statistic was lower than the $Z_{critical}$ for the established level of significance in only 10 tests, which led us to accept the null hypothesis H_{0A} for these occurrences. In the remaining 80 tests, the values of the Z-statistic showed significant deviations, which led to the acceptance of the alternative hypothesis H_{1A} for these digits. All the UGs showed significant deviations in at least 4 numbers. It is also shown in Table 5 that the results of the χ2 test reject the null hypothesis H_{0C}, which indicates that the distribution of the allotment bill values for all the UGs was not compatible with the probabilities predicted by the NB-Law. UG3 had the smallest deviation (129.16), and UG2 had the largest observed deviation (23,199.46).

Table 5 *The individual analysis of each management unit in state E2 applied to the 1st digit - Z-test and χ^2 test*

	Digit	UG1	UG2	UG3	UG4	UG5	UG6	UG7	UG8	UG9	UG10
Z-Test	1	0.891*	39.581	4.714	15.840	31.638	2.780	17.193	8.943	1.328*	9.023
	2	5.111	30.745	1.883*	12.240	19.552	6.961	2.573	10.841	2.514	10.295
	3	2.481	23.145	0.295*	4.420	2.630	6.936	3.647	19.766	3.997	11.973
	4	10.297	20.405	0.649*	11.873	16.702	3.938	6.825	3.998	9.767	12.086
	5	13.705	5.244	3.274	4.213	21.784	15.758	5.140	9.117	26.621	6.266
	6	2.745	5.127	5.990	8.481	13.970	3.040	9.413	6.257	8.583	11.912
	7	10.162	100.698	6.231	3.685	1.009*	1.984	2.772	11.450	1.036*	12.411
	8	8.002	105.926	3.206	6.079	6.345	6.543	7.503	7.090	4.777	1.820*
	9	0.315*	15.206	4.513	7.015	1.790	6.316	0.072*	7.133	8.481	8.644
χ^2-Test		462.51	23,199.46	129.16	624.10	2,068.13	423.67	435.76	845.88	922.78	794.26

(*) No significant difference at α = 0.05 with $Z_{critical}$ <= 1.96 and $\chi^2_{critical}$ <= 15.507.

After completing the analysis of the 1st digit, the 134,281 contracts issued by the 20 UGs were grouped to diagnose the overall spending behaviors. The results are shown in Table 6 and Figure 1.

From the information in Table 6, it is evident that the values obtained with the Z-statistic were above the $Z_{critical}$ for all digits, which allowed us to reject hypothesis H_{0A} for all occurrences. The largest deviations were observed in the numbers 7, 9, 6, and 8. Another notable observation was the sign of the observed deviation because there was a greater occurrence of the numbers 7 and 8, whereas there was a reduction for numbers 9 and 6 compared to the proportion predicted by the NB-Law.

By examining the pattern of distortions in the 1st digit, we can observe the probable influence of the spending limit in the bidding process provided by Federal Law n. 8,666/93 on public spending. Based on a spending limit for non-engineering purchases and services of R$8,000.00, the deviations suggest the occurrence of a possible substitution of values beginning with the numbers 6 and 9 for the numbers 7 and 8 in values equal to or lower than the established legal limit. Therefore, the UGs avoid the necessity to perform the bidding process for the case of the number 9. Moreover, the absence of observed occurrences for the number 6 would reflect the identical substitution motivated by maximizing the benefit of the contract.

According to Krakar and Zgela (2009), excess digits should be given greater attention through careful analysis of past events and possible reasons for their occurrence. Rare numbers generally do not deserve additional attention because they are only a reflection of the surplus of the other number.

The result of the $\chi 2$ test (3,214.32) again led to a rejection of the null hypothesis H_{0C} for the grouped data because there was no agreement between the observed distribution and probabilities predicted in the NB-Law at a 5% significance level.

Table 6 *The grouped analysis applied to the 1st digit for UGs in the two states in their entirety*

Digits	Observed Number	Expected Number	Proportion from NB-Law	Observed Proportion	Deviation	Z Value	χ^2
1	39,342	40,419	0.301	0.2930	-0.008	6.402	28.68
2	23,170	23,647	0.1761	0.1725	-0.004	3.413	9.62
3	15,845	16,772	0.1249	0.1180	-0.007	7.645	51.20
4	12,651	13,012	0.0969	0.0942	-0.003	3.324	10.01
5	11,409	10,635	0.0792	0.0850	0.006	7.816	56.32
6	8,099	8,983	0.0669	0.0603	-0.007	9.654	87.07
7	11,971	7,788	0.058	0.0891	0.031	48.827	2,246.32
8	7,631	6,875	0.0512	0.0568	0.006	9.352	83.09
9	4,163	6,150	0.0458	0.0310	-0.015	25.932	642.02
Total	134,281	134,281	1.0000	1.0000	0.000		3,214.32

(*) No significant difference at α = 0,05 with $Z_{critical}$ <= 1,96 and $\chi^2_{critical}$ <= 15,507,

In general, apart from the specific rigor of the significance levels, a simple observation of the distribution for numbers 1 to 9 for the 1st digit, as depicted in Figure 1, showed that the NB-Law is also applicable to the values of allotment bills for state public spending. This confirmed the results obtained in previous studies that portrayed the law's applicability to municipal public spending (Santos & Diniz, 2004; Santos, Diniz, & Ribeiro Filho, 2003; Santos, Diniz, & Corrar, 2005; Diniz, Corrar, & Slomski, 2010).

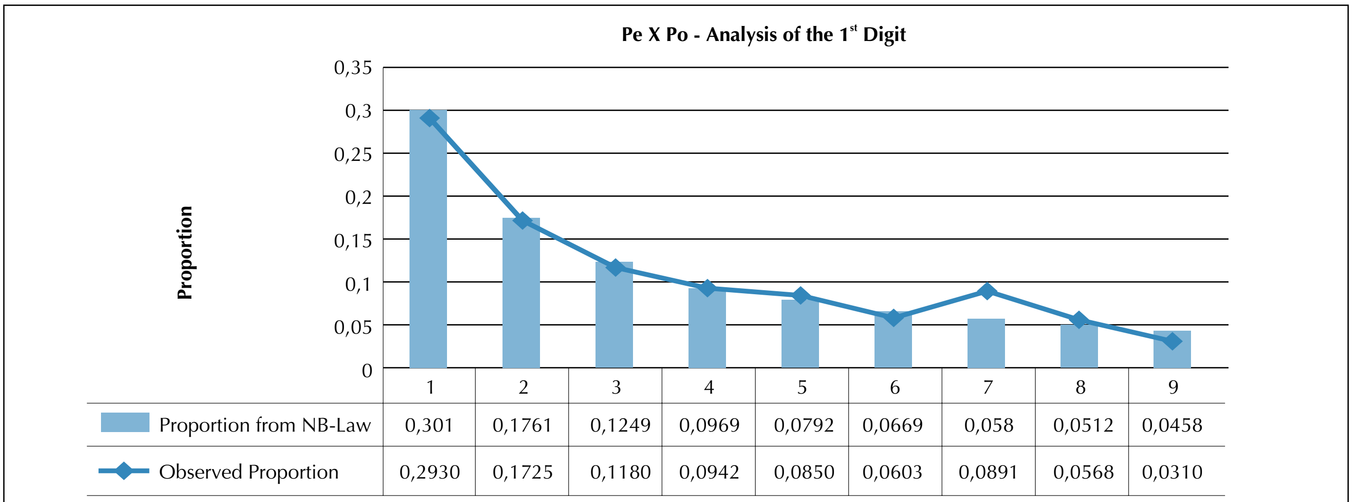

Figure 1 *The distribution of the occurrence of the 1st digit in the pooled data from the 20 UGs*

For the 2nd digit analysis, Tables 7 and 8 present the results of the Z-test and χ2 test applied to the distribution and observed occurrence of the numbers 0 to 9 in the 2nd digit compared to the values of the contracts issued by the 10 UGs in the sample for state E1.

Table 7 depicts the individual results for each UG and shows that the values obtained in 20 tests for the Z-statistic were lower than the $Z_{critical}$ for the established significance level. Therefore, we accepted the null hypothesis H_{0B} for these occurrences. In the 80 remaining tests, the results of the Z-statistic were higher than the $Z_{critical}$, which caused us to accept the alternative hypothesis H_{1B} for these values. All the UGs showed at least two numbers with significant deviations, from UG4 with deviations for numbers 0 and 8 to UGs 2, 8, and 10 with significant deviations in all 10 numbers analyzed. Based on the results obtained in the χ2 test, Table 7 shows the rejection of the null hypothesis H_{0D} for all the UGs. This result indicated non-conformity in the overall analysis of all numbers, in which UG4 had the smallest deviation (24.70) and UG2 had the largest (5,717.79).

Table 7 *The individual analysis of each management unit in state E1 applied to the 2nd digit - Z-test and χ^2 test*

Digit		UG1	UG2	UG3	UG4	UG5	UG6	UG7	UG8	UG9	UG10
Z-Test	0	6.753	52.680	43.378	2.448	69.740	31.413	29.978	22.819	20.001	38.453
	1	2.997	23.654	26.490	1.573*	6.632	13.306	10.631	10.364	7.694	5.544
	2	0.639*	14.008	4.286	1.032*	5.834	8.144	4.167	3.120	3.328	3.387
	3	2.558	21.552	20.827	1.593*	15.056	9.570	10.271	6.614	7.306	5.149
	4	2.525	16.081	2.365	1.792*	13.071	0.404*	0.694*	3.199	1.300*	5.337
	5	0.772*	35.741	22.635	1.054*	0.372*	5.423	3.143	7.231	4.213	5.849
	6	1.599*	3.781	6.317	1.693*	12.317	1.081*	1.136*	5.859	1.568*	6.110
	7	5.611	17.144	11.741	1.547*	11.114	12.539	8.504	6.585	5.315	9.082
	8	3.511	13.582	3.156	2.002	11.936	14.696	2.459	2.254	2.879	4.946
	9	0.299*	19.437	1.785*	0.820*	13.418	14.601	6.216	5.147	5.957	8.544
χ^2-Test		104.71	5,717.79	3,327.41	24.70	5,260.70	1,733.88	1,122.01	761.52	552.96	1,622.06

(*) No significant differences at α = 0.05 with $Z_{critical}$ <= 1.96 and $\chi^2_{critical}$<= 16.919.

Table 8 shows the results of the Z-test and χ2 test applied to the analysis of the 2nd digit for the 10 UGs in the sample from state E2.

Table 8 shows that the values obtained for the Z-statistic were lower than the $Z_{critical}$ in only 8 tests, and the null hypothesis H_{0B} was accepted for these observations. For the remaining 92 tests, the values of the Z-statistic indicated significant deviations, which led us to accept the alternative hypothesis H_{1B} for these values. All of the UGs showed deviations in at least 4 numbers. Table 8 also shows the results of the χ2 test, which indicated a rejection of the null hypothesis H_{0D} for all UGs. This result indicated that the distribution of the values for the allotment bills was not compatible with the probabilities expected by the NB-Law, in which UG6 had the smallest deviation (152.45) and UG2 had the largest (20,035.18).

Table 8 *The individual analysis for each management unit in state E2 applied to the 2nd digit - Z-test and χ^2 test*

Digit		UG1	UG2	UG3	UG4	UG5	UG6	UG7	UG8	UG9	UG10
Z-Test	0	83.257	138.594	15.571	11.882	7.311	3.683	4.477	21.128	2.874	12.282
	1	29.110	24.306	7.597	12.081	16.455	7.130	13.055	9.272	3.438	5.095
	2	6.457	22.412	4.328	4.860	10.877	3.429	0.679*	7.598	4.933	7.589
	3	11.724	23.483	9.581	12.722	47.712	4.669	29.029	9.133	9.959	13.331
	4	20.267	24.752	5.172	9.197	10.309	2.926	5.314	6.552	8.854	6.832
	5	20.999	9.746	4.263	3.865	14.466	5.680	6.187	25.325	1.191*	1.729*
	6	0.964*	22.786	3.356	1.866*	14.624	0.271*	2.503	8.042	7.928	9.355
	7	18.664	16.192	1.603*	16.816	16.604	0.882*	8.892	3.366	10.991	39.474
	8	17.425	13.498	2.602	3.780	14.577	3.985	4.834	3.450	4.613	12.467
	9	11.411	12.260	6.059	13.347	10.891	3.590	10.742	6.773	2.829	6.527
χ^2-Test		8,499.74	20,035.18	458.18	948.91	3,469.54	152.45	1,192.09	1,339.58	396.41	2,094.47

(*) No significant difference at α = 0.05, with $Z_{critical}$ <= 1.96 e $\chi^2_{critical}$ <= 16.919.

The 134,281 contracts in the sample were grouped to assess the overall spending behavior to conclude the analysis of the 2nd digit; the distribution and observed deviations are shown in Figure 2 and detailed in Table 9.

The data in Table 9 shows that the only number to show an insignificant deviation for the Z-statistic was 2. All the other numbers showed significant variations; therefore, we accepted the null hypothesis H_{0B} for the number 2 and rejected it for the other numbers. Only the numbers 0 and 5 among those that exceeded the $Z_{critical}$ showed an increased occurrence compared to the proportion predicted by the NB-Law, which again indicated that the sign of the deviation was a determining factor in the analysis. This pattern of observed deviations with excess occurrences for the numbers 0 and 5 in the 2nd digit reflected the higher occurrence of rounded values in the contracts, such as 4,000, 250, and 8,000. Among the possible factors that explained this occurrence, it is noteworthy that the inclusion of fictitious values or contracted prices are not formed because of a direct application of a profit margin to the amount of costs and expenses in producing and selling. The necessity of qualitative analyses, through auditing practices, on the history of events and possible cyclical factors should again be emphasized to determine whether these deviations arose from errors and fraud or could be explained by behavioral and/or normative factors.

The excess number of occurrences for the number 0 was also observed in previous studies conducted by Carslaw (1988) and Krakar and Zgela (2009), which applied the analysis of the 2nd digit to financial data on revenue and financial transactions in the banking system. The result obtained in the $\chi 2$ test (24,060.16) confirmed the rejection of the null hypothesis H_{0C} because the distribution in the overall analysis did not conform with the probabilities predicted by the NB-Law for the 2nd digit at a 5% significance level.

Table 9 *The grouped analysis applied to the 2nd digit - UGs in the two states in their entirety*

Digit	Observed Number	Expected Number	Proportion from NB-Law	Observed proportion	Deviation	Z-Value	χ^2
0	33,603	16,060	0.1196	0.2502	0.131	147.529	19,162.92
1	10,488	15,295	0.1139	0.0781	-0.036	41.284	1,510.56
2	14,569	14,610	0.1088	0.1085	0.000	0.353*	0.11
3	11,404	14,006	0.1043	0.0849	-0.019	23.223	483.23
4	9,994	13,468	0.1003	0.0744	-0.026	31.558	896.27
5	14,639	12,985	0.0967	0.1090	0.012	15.268	210.69
6	10,548	12,542	0.0934	0.0786	-0.015	18.694	316.97
7	10,316	12,139	0.0904	0.0768	-0.014	17.344	273.77
8	10,922	11,763	0.0876	0.0813	-0.006	8.113	60.13
9	7,798	11,414	0.0850	0.0581	-0.027	35.378	1,145.50
Total	134,281	134,281	1.0000	1.0000	0.000		24,060.16

(*) No significant difference for $\alpha = 0.05$ with $Z_{critical} <= 1.96$ and $\chi^2_{critical} <= 16.919$.

Figure 2 visually shows the excess proportion of observations for the number (0), 25.02% of occurrences, compared to the 11.96% proportion expected from the NB-Law.

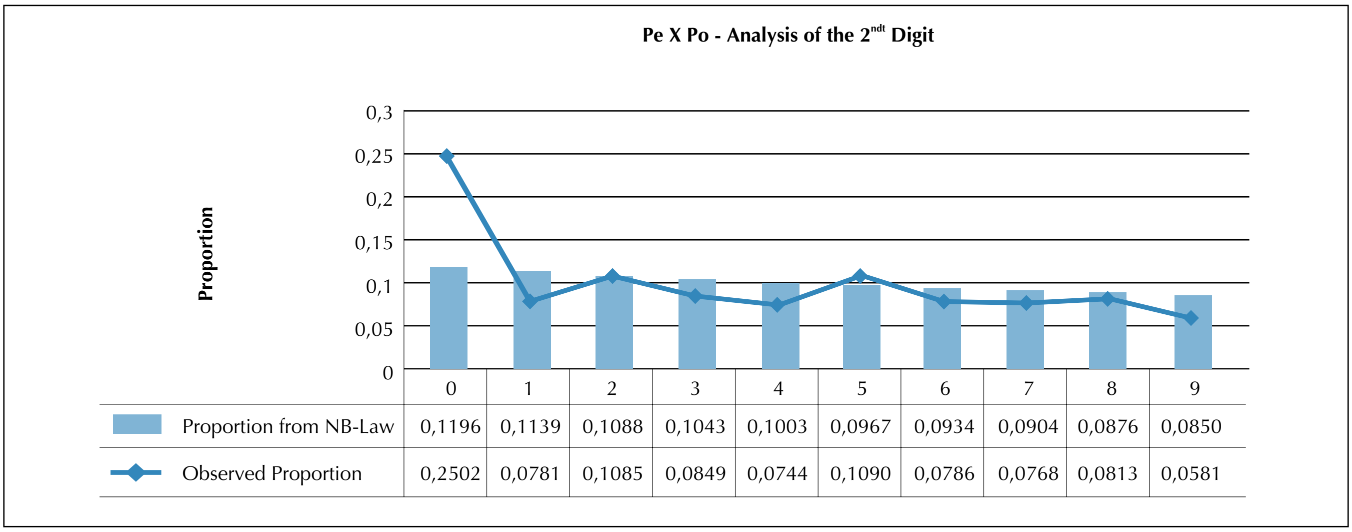

Figure 2 *The distribution of occurrences for the 2nd digit using pooled data from the 20 UGs*

In general, apart from the specific rigor of the established significance level, simple observations from Figures 1 and 2 of the digit distributions show that the NB-Law was also applicable to the values of spending bills from federal public entities, including those at the state level.

5 FINAL CONSIDERATIONS

This study evaluated the occurrence of significant deviations in the distribution of the 1st and 2nd digits of state public spending compared to the behavior predicted by the NB-Law to identify accounting models that apply to auditing the public sector. The following was confirmed by the study results:

- The significant deviations in the individual analyses for the UG and state groupings compared to the distribution predicted by the NB-Law for the 1st digit. The analyses grouped by state showed that the largest deviations occurred for spending bills beginning with 7 and 8, for which there were excess occurrences, and with 9 and 6, with scarce occurrences. This pattern of deviation suggests behaviors to avoid the bidding process in public spending. Such avoidance behavior would not necessary be positively associated with fractioning expenses. Based on the legal imposition of conducting the bidding process under the law for values above R$8,000.00, this avoidance behavior could be explained by the administrator's preference, in exercising discretion, to avoid contracts for values near and above the bidding threshold.
- Significant deviations occurred in the individual analyses for the UGs and state groupings compared to the distribution predicted by the NB-Law for the 2nd digit. The analyses grouped by state showed the occurrence of significant positive deviations, or excess occurrences, for bills with the numbers 0 and 5 in the 2nd digit. This pattern of deviation indicated the occurrence of rounding in the contract prices, which apparently were not formed from directly applying a profit margin to the amount of costs and expenses of producing and selling.

Therefore, in response to the 1st hypothesis in this article for the local analysis of deviations in the 1st digit (Z-test), we accepted the null hypothesis H_{0A} for 25 tests applied to state E1 (Table 4) and 10 tests applied to state E2 (Table 5). We rejected this hypothesis for the other tests. With regard to the analysis grouped by states, we rejected the null hypothesis H_{0A} for all of the numbers tested (Table 6).

In the 2nd hypothesis, which addressed the local analysis of deviations applied to the 2nd digit (Z-test), we accepted the null hypothesis H_{0B} for 20 tests applied to state E1 (Table 7) and 8 tests applied to state E2 (Table 8). We rejected this hypothesis for the other tests. With regard to the analysis grouped by states, we accepted H_{0B} for the number 2 and rejected it for the other numbers (Table 9).

For the 3rd hypothesis, which addressed the overall analysis of deviations applied to the 1st digit ($\chi 2$ test), we rejected the null hypothesis H_{0C} for all 10 UGs in state E1 (Table 4) and state E2 (Table 5). We also rejected H_{0C} for the analyses grouped by states (Table 6). The overall analysis applied to the 2nd digit, established in the 4th and final hypothesis, similarly indicated the rejection of the null hypothesis H_{0D} for all 20 UGs tested (Tables 7 and 8) and for the analysis grouped by state (Table 9).

The use of the NB-Law as a methodology for application to auditing in the public sector has been shown to be effective in determining deviations in spending behavior. Through quantitative analysis of the history of events and possible causes of these deviations, we can detect the occurrence of errors, fraud, and behavioral tendencies of managers in using public resources.

Therefore, we observed that the law's use is an effective tool for auditing teams, particularly in the preparation of planning and determining the auditing sample. The law's application, coupled with a digital auditing environment, will enable external control bodies to issue automatic alerts indicating deviations and the excess occurrence of numbers. This type of alert would be an indicator to support managerial decisions in auditing teams with repercussions in optimizing the results through allocating resources to prioritized areas or entities.

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