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Cryptocurrency price returns volatility modeling and forecasting with GARCH models

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Abstract

Purpose – The paper aims to identify suitable conditional variance models for the estimation and forecasting of cryptocurrency returns volatility.

Design/methodology/approach – The methodology comprises the use of GARCH-family models estimated by maximum likelihood considering different scedastic functions, number of parameters and error distributions. A cross-validation approach is conducted under different market dynamics to provide robust results.

Findings – Results indicated that the best GARCH methods for digital coins volatility modeling and forecasting are those associated with a small number of parameters, allowing for asymmetric volatility behavior and considering normal/student distributions.

Research limitations/implications – The findings indicated that volatility behaves differently for each evaluated cryptocurrency, and the selection of the best scedastic function depends on the corresponding digital coin more than the period under evaluation.

Practical implications – Investors should prefer parsimonious GARCH structures when modeling and forecasting cryptocurrency volatility, and must consider the current state of the market as the methods lose accuracy in high-volatile periods.

Social implications – The work provides a better understanding of the volatility dynamics of cryptocurrencies, providing evidence of more accurate tools for risk management in this volatile market. Further, better-informed investors on the risks associated with this market are less susceptible to high price variations.

Originality/value – The research presents an extensive experimental study to identify the optimal GARCH structure for modeling and forecasting return volatility in digital currencies, considering various market conditions and digital coins, which yields more robust results.

Keywords Cryptocurrencies, Volatility, GARCH models, Forecasting

Paper type Research paper

1. Introduction

Since the creation of Bitcoin (Nakamoto, 2008), as the first decentralized digital currency, cryptocurrencies have gained significant importance in the global financial market. According to CoinMaketCap, the global crypto market capitalization is over USD 1.26

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trillion as of April 2023. Digital coin markets also grow through the creation of new financial instruments. For instance, BitMEX, a crypto trading platform, introduced perpetual crypto futures contracts in 2016; the Chicago Mercantile Exchange promoted the negotiation of Bitcoin futures in 2017; and more recently, ProShares, an issuer of exchange-traded funds (ETFs), launched the first Bitcoin-associated ETF in 2021. Option crypto-based contracts are also a growing market (Söylemez, 2019), revealing the higher relevance of cryptocurrencies in financial markets.

As market participants increasingly engage in trading digital coins, the impact of speculative activities on price dynamics becomes a concern (Corbet, Lucey, & Yarovaya, 2018a; Blau, 2017; Baek & Elbeck, 2015). For instance, the work of cheah2015speculative provided evidence that the Bitcoin dynamic is affected by speculative movements (bubbles). In addition, one of the main features of the digital coin market is the high volatility dynamics, where prices change considerably over time, in comparison to traditional assets such as stocks. It offers investors significant profit possibilities in a riskier environment.

Given the high price variations of cryptocurrencies, the selection of an adequate modeling technique for returns volatility estimation plays a crucial role. Volatility is a key variable for risk assessment, portfolio composition, risk management and loss monitoring. As volatility is not an observed variable, its estimation is of fundamental interest. Further, as financial asset returns present conditional variance as a stylized fact, the modeling framework must address the time-varying aspect of the return variance, using robust financial econometric time series techniques (Su, 2021).

Hence, this paper evaluates whether GARCH-family (Generalized Autoregressive Conditional Heteroskedasticity) models are suitable for cryptocurrency volatility modeling and forecasting. In addition, the work aims to identify the best GARCH specification for digital coins conditional variance estimation. The methodology includes an extensive empirical study for a sample of leading cryptocurrencies (Bitcoin, Dogecoin, Ethereum, Litecoin, Stellar and Ripple) for the period from January 2016 to December 2021. Different GARCH models are estimated in terms of the scedastic function, the number of parameters, and the error distribution specification. A cross-validation strategy, considering three out-of-sample sets under different market dynamics, is also conducted to promote more robust results.

The contributions of this work are as follows. First, it verifies whether GARCH family models are also appropriate for modeling the conditional variance of high volatility assets such as cryptocurrencies. Second, it provides an extensive experimental study to detect the most appropriate GARCH structure to model and forecast return volatility for digital currencies in general, which provides investors with an accurate modeling approach for their risk management strategies when trading cryptocurrencies. Finally, the work provides findings to the literature on the analysis of cryptocurrency volatility dynamics, considering alternative digital coins and not focusing solely on Bitcoin. Indeed, Corbet, Meegan, Larkin, Lucey, and Yarovaya, (2018b) pointed out that some cryptocurrencies have particular dynamics due to their corresponding technical mechanisms; thus, investors must consider this aspect to benefit from diversification when composing portfolios.

The paper is organized as follows. Section 2 addresses a brief literature review. The methodology is provided in Section 3. Section 4 comprises the results and corresponding discussions. Finally, Section 5 summarizes the findings and lists future research alternatives.

2. Literature review

The literature concerning risk management in the cryptocurrency market is still under development. Phillip, Chan, and Peiris (2018) investigated the volatility stylized facts of 224 cryptocurrencies. The authors indicated that return volatility is generally long memory persistent, presents clustering volatility and heavy tail characteristics. Similarly, Ma and Tanizaki (2019) and Kinateder and Papavassiliou (2019) found the existence of calendar anomalies on the return and volatility of Bitcoin. Return and volatility properties of non-fungible tokens and cryptocurrencies are also recently discussed in Ghosh, Bouri, Wee, and Zulfiqar (2023).

The relation between conditional variance and market sentiment for Bitcoin was studied by López-Cabarcos, Pérez-Pico, Pi˜neiro-Chousa, and ˇSevíc, (2019). A GARCH model with exogenous variables provided evidence of the influence of investor sentiment on Bitcoin volatility. López-Cabarcos et al. (2019) related Bitcoin to stock market dynamics. The authors indicated that Bitcoin provides safe-haven features in periods of high volatility, whereas it is more prone to speculative purposes when the market is less volatile. Wang, Ma, Bouri, and Guo (2023) showed that macroeconomic and technical indicators promote relevant information for forecasting Bitcoin realized volatility.

Concerning the use of GARCH-family models, Katsiampa (2017) evaluated the selection of the best method to describe Bitcoin volatility returns adequately. With data from July 2010 to October 2016, the authors showed that the standard GARCH approach performed adequately. They also indicated that allowing for short and long-run components improves Bitcoin volatility modeling. Chen, Huang, and Liang (2023) indicated that skewed and heavy-tailed distribution contribute to better performance in forecasting risk measures for cryptocurrencies using GARCH-family models; however, they do not improve the accuracy of volatility forecasting.

In a multivariate framework, Katsiampa (2019) evaluated the joint volatility dynamics of Bitcoin, Ether, Ripple, Litecoin and Stellar Lumen. A multivariate asymmetric GARCH model was considered to estimate conditional covariances. Their findings indicated asymmetric volatility effects for good and bad news, and significant volatility co-movements between cryptocurrencies were found.

Aras (2021) forecasted Bitcoin volatility by combining GARCH models and machine learning techniques. For the period from July 2013 to August 2020, when compared to traditional GARCH approaches, the ensemble method proposed was able to produce more accurate volatility forecasts. Similarly, Peng, Albuquerque, de S´a, Padula, and Montenegro, (2018) estimated the volatility of Bitcoin, Ethereum, Dash and standard currency exchange rates in an ensemble framework composed of GARCH and Support Vector Regression techniques.

More recently, Bergsli, Lind, Moln´ar, and Polasik, (2022) compared GARCH models with heterogeneous autoregressive methods (HAR), which are capable of capturing the observed persistence in the realized variance. For Bitcoin data, empirical results indicated that the HAR models outperformed the GARCH approach for intraday data. For daily data, GARCH methods provided a higher accuracy. Alternatively, Sun and Kristoufek (2024) compared the modeling of cryptocurrency volatility using approaches from the GARCH-family and range-based models, which use information from maximum and minimum prices.

Generally, literature provides evidence on the ability of GARCH-family models to model and forecast the conditional variance dynamics of cryptocurrencies (Fakhfekh & Jeribi, 2020; Fung, Jeong, & Pereira, 2022; Köchling, Schmidtke, & Posch, 2020). Particularly, Katsiampa (2017), Aras (2021) and Bergsli et al. (2022) focus solely on Bitcoin. Chen et al. (2023) concentrate on analyzing probability distributions in conditional variance models, while also restricting their analyses to Bitcoin and Ethereum. In the current paper, our contribution is to offer a more comprehensive approach by evaluating a broader range of cryptocurrencies (Bitcoin, Dogecoin, Ethereum, Litecoin, Stellar and Ripple), exploring different conditional variance structures (scedastic functions), considering various probability distributions and distinct out-of-sample periods dynamics.

3. Methodology

This paper proposes an extensive experimental analysis to select the best GARCH models for cryptocurrency volatility modeling and forecasting. GARCH-family models were considered as they account for financial asset return stylized facts, such as conditional variance behavior, volatility clustering and asymmetric volatility (Katsiampa, 2019). Computational experiments include the evaluation of different scedastic functions, innovation distributions and model structure (number of parameters).

3.1 GARCH-family models

GARCH-family models describe the dynamics of asset price returns as a conditional time-varying process. It considers price log-returns, $y_t \in \Re$, calculated as $y_t = ln(P_t) - ln(P_{t-1})$, where P_t represents the price at time t. It is assumed that $E[y_t] = 0$ (zero mean), and the absence of serial correlation in $\{y_t\}$. An autoregressive process is estimated to remove autocorrelation. Hence, generally, a GARCH model can be described as:

$$y_t| \sim \mathcal{D}(0, h_t, \xi), \quad (1)$$

Where $\mathcal{D}(0, h_t, \xi)$ is a continuous distribution; h_t represents the conditional variance at time t, and $\boldsymbol{\xi}$ indicates shape parameters according to the innovation distribution (for instance, tail and asymmetry features). Standardized innovations are represented by $y_t \equiv h_t^{1/2}\eta_t$, with $\eta_t \sim i.i.d.\,\mathcal{D}(0, 1, \xi)$.

In the GARCH modeling framework, $h_t \equiv h(y_{t-1}, h_{t-1})$ is assumed to depend on past returns and past conditional variance. This dependence is measured by the corresponding scedastic specifications, which define the form of $h(\cdot)$. This paper assumes four different scedastic functions: the standard GARCH; the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH), the Exponential GARCH (EGARCH) and the Component GARCH (CGARCH).

A GARCH(p, q) model (Bollerslev, 1986) can be described as:

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j y_{t-j}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}, \quad (2)$$

Where p and q represent the number of lagged variances and squared returns (shocks), respectively; α_0, α_j, β_i are the parameters to be estimated, $i = 1, \ldots, p$ and $j = 1, \ldots, q$; and $\alpha_0 > 0$, $\alpha_i \geq 0$, and $\beta_i \geq 0$ represent the parameters constraints.

The GARCH model does not distinguish asymmetrical effects of past returns on the dynamic of the conditional variance. Hence, negative and positive past returns, as representatives of bad and good news, respectively, have the same impact on the conditional variance. To overcome this limitation, the GJR-GARCH model, developed by Glosten 1993 relation, allows for asymmetric effects. A GRJ-GARCH model (p, q) can de defined as:

$$h_t = \sum_{j=1}^{q} \left(\alpha_j y_{t-j}^2 + \gamma_j I_{t-j} y_{t-j}^2 \right) + \sum_{i=1}^{p} \beta_i h_{t-i}, \quad (3)$$

Where $I_{t-j} = 1$ if $y_{t-j} < 0$ (negative past return or bad news), and $I_{t-j} = 0$ if $y_{t-j} \geq 0$ (positive return or good news). The parameter γ_j captures the asymmetry effect. If these parameters are statistically significant, there is a distinct impact on the conditional variance process of positive and negative returns (leverage effect). Parameters constraints in the GJR-GARCH model are $\alpha_0 > 0, \alpha_i \geq 0, \beta_i \geq 0, / \alpha_i + \gamma_i \geq 0$.

Similar to the GJR-GARCH model, the EGARCH model, proposed by Nelson 1991 conditional, also takes into account asymmetric effects, but with no restrictions on the positivity for the estimated parameters. A general EGARCH(p, q) model is described by the following structure:

$$ln(h_t) = \sum_{j=1}^{q} \left(\alpha_j y_{t-j} + \gamma_j \left(|\nu_{t-j}| - E|\nu_{t-j}|\right)\right) + \sum_{i=1}^{p} \beta_i ln(h_{t-i}), \tag{4}$$

Where γ_j is the parameter associated with asymmetry effects on the conditional variance process.

Finally, this paper considers the Component GARCH (CGARCH), proposed by Engle 1999 cointegration. The model describes conditional volatility by combining a permanent component and a transitory component. A CGARCH(p, q) model is described by:

$$h_t = q_t + \sum_{j=1}^{q} \alpha_j \left(y_{t-j}^2 - q_{t-j}\right) + \sum_{i=1}^{p} \beta_i (h_{t-i} - q_{t-i}), \tag{5}$$

Where $q_t = \omega + \rho q_{t-1} + \phi\left(y_{t-1}^2 - h_{t-1}\right)$, q_t represents the permanent component, and $h_{t-j} - q_{t-j}$ is the transitory component, defined as the difference between the conditional variance and its trend. The model allows the intercept of the GARCH model to be time-varying following first-order autoregressive-type dynamics. ω, ρ ($\rho < 1$) and ϕ are the permanent component parameters.

For the four models, GARCH, GJR-GARCH, EGARCH and CGARCH, the corresponding orders p and q are selected according to the Bayesian information criterion (BIC) for different structures. In addition, the error process ν_t can assume the following distributions: normal distribution (norm), skewed normal distribution (snorm), student's t distribution (std), skewed student's t distribution (sstd), generalized error distribution (ged) and the skewed generalized error distribution (sged). Skewed distributions generalize standard distributions to allow for non-zero skewness, which is a common feature of financial price returns.

Overall, for each cryptocurrency, model setting includes 216 different specifications recovered as combinations of:

- four conditional variance specifications (scedastic function), GARCH, GJR-GARCH, EGARCH and CGARCH;
- 72 different parametrizations considering the combinations for $p \in [0, 3]$ and $q \in [0, 3]$; and
- and six distinct conditional distribution (norm, snorm, std, sstd, ged and sged).

For the six digital coins considered, under these different possibilities for model structure modeling, a total of 648 GARCH-family models are estimated. Models are estimated by maximum likelihood. All experiments were performed in R software with the use of the "rugarch" package.

3.2 Performance metrics

Methods are evaluated for in-sample and out-of-sample sets. In-sample analysis is based on BIC criteria to indicate the best model in terms of parsimony (higher fit to data and lower number of parameters). Otherwise, the out-of-sample set is used to evaluate the forecasting capability of the models concerning the horizon of one-step-ahead. The quality of forecasts is measured in terms of accuracy, calculated from different loss functions. This work measures forecasting accuracy using the mean square error (MSE), the mean absolute error (MAE), the mean absolute percentage error (MAPE) and the QLIKE function, calculated as, respectively (Patton, 2011):

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(h_t - \widehat{h}_t\right)^2, \quad (6)$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|h_t - \widehat{h}_t\right|, \quad (7)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{h_t - \widehat{h}_t}{h_t}\right|, \quad (8)$$

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T}\left(ln\left(\widehat{h}_t\right) + \frac{h_t}{\widehat{h}_t}\right), \quad (9)$$

Where $\widehat{h}_t$ is the volatility forecast at t, obtained by the GARCH approaches, h_t is the actual volatility at t, and T is the out-of-sample size. As volatility is an unobserved variable, we used $h_t = y_t^2$, i.e. the squared returns, as a proxy of actual volatility. For all the accuracy measures, lower values are associated with a better predictive performance.

4. Experimental analyzes

4.1 Data

This paper considered a total of six cryptocurrencies selected based on their market capitalization (the leading cryptos) and the availability of considerable daily historical data. The digital coins are: Bitcoin (BTC), Dogecoin (DOGE), Ethereum (ETH), Litecoin (LTC), Stellar (XLM) and Ripple. Daily closing prices in USD were collected using the Coingecko API [1]. The platform provides average trading prices of each cryptocurrency for different exchanges. Prices are weighted according to the corresponding trading volumes.

Data ranges from January 1, 2016, to December 31, 2021, within a total of 2,190 daily price observations. The sample begins in 2016, when the number of cryptocurrencies was low, and liquidity and historical information were more readily available. This period was chosen for the analysis to ensure a sufficient amount of information for estimating the models. The sample ends in 2021, as this was the last available data information when the research was performed. The six digital coins were selected as they were the most liquid cryptos with historical information for the period range considered. Figure 1 displays the temporal evolution of cryptocurrency prices and the corresponding return series.

Table 1 provides summary statistics of cryptocurrency returns. Returns are generally zero mean and exhibit high volatility (high standard deviation). These significant price fluctuations can be observed by the range between minimum and maximum returns values. According to Table 1, returns are generally right-skewed and have high values of kurtosis. A higher kurtosis is associated with fat-tailed distributions. Descriptive statistics of the S&P

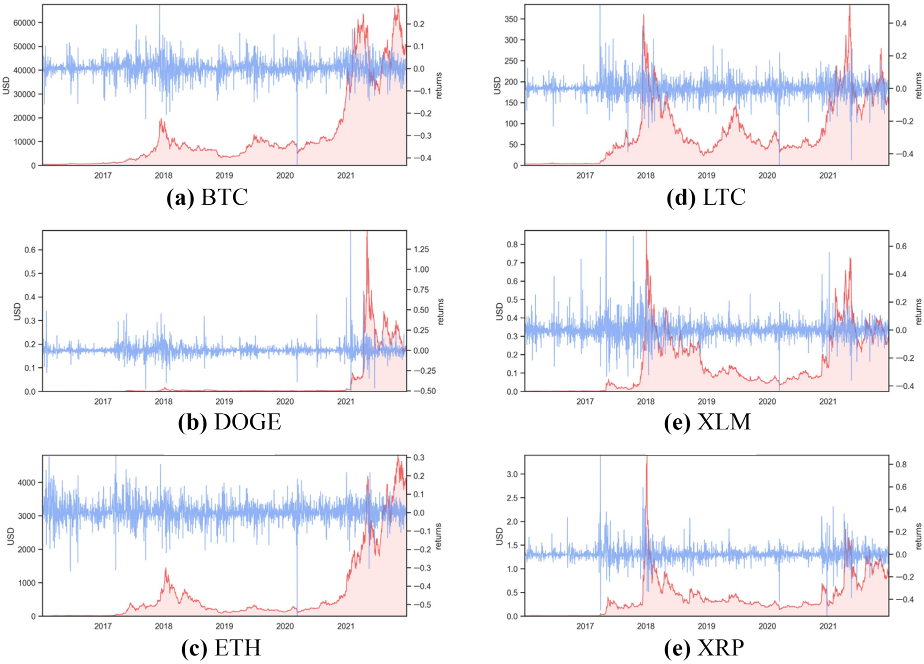


Figure 1. Daily prices and returns for the selected cryptocurrencies for the period from January 2016 to December 2021
Source(s): Authors' own work

Table 1. Summary statistics of cryptocurrency daily returns for the period from January 2016 to December 2021

Crypto	Mean	Median	Min.	Max.	SD	SD (% p.a.)	Variance	Skewness	Kurtosis
Bitcoin	0.002	0.002	−0.434	0.287	0.040	63	0.002	−0.632	10.220
Dogecoin	0.003	−0.001	−0.507	1.479	0.077	122	0.006	4.051	69.379
Ethereum	0.003	0.002	−0.563	0.312	0.058	92	0.003	−0.432	7.595
Litecoin	0.001	0.000	−0.471	0.514	0.056	89	0.003	0.226	11.773
Ripple	0.002	−0.001	−0.550	0.881	0.068	108	0.005	1.698	24.739
Stellar	0.002	−0.001	−0.439	0.712	0.074	117	0.005	1.750	16.900
S&P 500	0.001	0.001	−0.128	0.090	0.012	18	0.000	−1.204	23.835

Source(s): Authors' own work

500 index returns for the same period are also presented, where the higher volatility of digital currency returns can be observed (see Table 1) – especially concerning the percent per annum standard deviations (SD % p.a.) values.

For cross-validation and robustness purposes, the whole sample was divided into three in-sample/out-of sample sets as detailed in Table 2. Each set comprised three years of daily data. For each period, the first two years (in-sample) were used for model estimation and the

Table 2. In-sample and out-of-sample sets starting and ending dates considered for the modeling and forecasting of cryptocurrency volatility

In-sample set			Out-of-sample set		
Start date	End date	# obs.	Start date	End date	# obs.
1/02/2016	12/31/2018	1,094	1/01/2019	12/31/2019	364
1/01/2017	12/31/2019	1,094	1/01/2020	12/31/2020	365
1/01/2018	12/31/2020	1,095	1/01/2021	10/31/2021	304

Source(s): Authors' own work

last year was used for the forecasting analysis (out-of-sample). Each out-of-sample period reflects a different market dynamic and thus provides robustness in the evaluation of the volatility forecasts.

4.2 In-sample volatility modeling results

For each cryptocurrency return series, the best GARCH, GJR-GARCH, EGARCH and CGARCH model were selected according to their corresponding BIC value. Table 3 provides the results for each in-sample period. Due to length limitations, we have described the best results here. Nevertheless, all the estimated models parameters and metrics are available upon request. Generally, the selected models and error distributions do not differ among the cryptocurrencies, regardless of the in-sample period. Most models presented a student distribution, except for Ethereum (ETH), which presented a GED distribution for all periods as the best selection in terms of BIC (see Table 3).

Table 3. GARCH models selected specifications for cryptocurrency volatility modeling

Crypto	In-sample period	Model	Distribution	BIC
BTC	2016–2018	EGARCH(1,2)	sstd	−4.0881
	2017–2019	EGARCH(3,3)	sged	−3.7722
	2018–2020	EGARCH(1,1)	std	−4.0481
DOGE	2016–2018	EGARCH(1,2)	std	−3.2035
	2017–2019	EGARCH(1,1)	std	−3.1996
	2018–2020	CGARCH(1,1)	std	−3.6417
ETH	2016–2018	GARCH(1,1)	ged	−2.9392
	2017–2019	GARCH(1,1)	ged	−3.1857
	2018–2020	GARCH(1,1)	ged	−3.3790
LTC	2016–2018	EGARCH(1,1)	std	−3.6548
	2017–2019	CGARCH(1,1)	std	−3.0801
	2018–2020	GARCH(1,1)	std	−3.2583
XLM	2016–2018	EGARCH(1,1)	std	−2.5962
	2017–2019	CGARCH(1,1)	std	−2.7332
	2018–2020	CGARCH(1,1)	std	−3.1976
XRP	2016–2018	EGARCH(2,1)	std	−3.3367
	2017–2019	CGARCH(1,1)	ged	−3.2075
	2018–2020	CGARCH(1,1)	std	−3.5373

Note(s): The selected scedastic function, the structure, the error distribution, and the corresponding value of the BIC information criteria are given for each crypto and each in-sample period
Source(s): Authors' own work

It is interesting to note that model selection is related to the behavior of each series, instead of being dependent on the period. In other words, Table 3 indicates that the same GARCH structure (in terms of scedastic function) is suitable for different market dynamics, but depends on the corresponding digital coin generating process.

The best volatility modeling for BTC and DOGE is associated with the EGARCH model, which uses student or skewed student distributions (see Table 3). For ETH, the GARCH model with GED distribution showed the best performance in the three in-sample periods. On the other hand, for Litecoin (LTC), the best scedastic function depends on the period, but with a student distribution it is kept the same. For Ripple (XRP) and XLM, the best model for the first period was the EGARCH with student distribution, while for the other two in-sample periods, the CSGARCH model with student distribution remains as the best volatility modeling framework. Finally, according to Table 3, most of the models with the best fit are parsimonious, i.e. considered a simpler (1) structure.

4.3 Out-of-sample volatility forecasting results

For each cryptocurrency and loss function (MSE, MAE, MAPE and QLIKE), the best predictive model was selected for each out-of-sample set (2019, 2020 and 2021). Results are reported in Table 4.

According to Table 4, the better models were EGARCH and CGARCH, which are associated with the lowest values of the loss functions and allow series to be modeled for particular contexts. For the case of EGARCH, the conditional variance takes into account asymmetric effects, where positive and negative returns have a different impact on the conditional variance. In addition, the CGARCH divides variance into permanent and transitory components, allowing shocks to be transitory and distinguishing short-term from long-term volatility.

Regarding the MSE metric, the EGARCH and CGARCH models were the ones that presented the best results (see Table 4). The most common distributions were the Normal distribution (norm) and its asymmetrical versions (snorm). Except for XRP, all digital coins selected the same GARCH model structure for consecutive out-of-sample periods. Table A1 in Appendix provides the descriptive statistics of the forecast errors. Generally, the errors have an approximately zero mean and similar standard deviations.

Regarding the MAE loss function, CGARCH also obtained suitable results, with the most frequent distributions being the normal and its asymmetric version in first place, and the asymmetric GED (sged) and student's t distributions as runners up (Table 4). Particularly for Litecoin (LTC), the three out-of-sample selected the CGARCH structure as the one associated with the higher forecasting accuracy. MAPE loss function presented results quite similar to MSE, but differing by suggesting the better distributions as normal, student's t and GED in their standard versions (see Table 4).

Table 4 also indicates that the EGARCH models have the highest number of occurrences concerning the QLIKE loss function. This structure is more associated with the student and asymmetric student distributions, followed by the normal distribution.

For a more general analysis, Tables 3 and 4 also indicate that the models and error distributions are consistently the same among the BIC criteria and the loss functions. Nevertheless, for each cryptocurrency and out-of-sample set the model and error distribution is particular, which means that the volatility generation process is dependent on the digital coin and on the current state of the market.

For all cryptocurrencies, variations of the standard GARCH model provided a better forecasting capability (see Table 4). The exception is DOGE, which is associated with an accurate forecasting ability for the standard GARCH structure.

Table 4. GARCH models selected specifications for cryptocurrency volatility forecasting

Crypto	Period	Model	Dist.	MSE	Model	Dist.	MAE	Model	Dist.	MAPE	Model	Dist.	QLIKE
Btc	2019	CGARCH(1,1)	norm	9.7E-06	CGARCH(1,1)	sstd	0.0016	GJRGARCH(3,2)	sstd	1427.2	EGARCH(2,1)	sstd	−8.2183
	2020	EGARCH(1,1)	sged	1.0E-04	CGARCH(3,3)	sstd	0.0020	EGARCH(3,3)	std	60911.6	GJRGARCH(2,3)	norm	17.9420
	2021	EGARCH(3,3)	snorm	1.0E-05	GJRGARCH(1,2)	snorm	0.0019	EGARCH(3,3)	norm	405.9	EGARCH(1,1)	sstd	−10.0134
DOGE	2019	EGARCH(1,2)	sged	9.2E-06	CGARCH(1,3)	norm	0.0014	CGARCH(2,1)	norm	2036.8	GJRGARCH(3,2)	norm	−6.2956
	2020	EGARCH(1,1)	norm	1.8E-04	CGARCH(2,1)	norm	0.0035	GJRGARCH(3,3)	std	8898.2	GARCH(3,3)	norm	5.9386
	2021	GARCH(1,2)	snorm	1.6E-02	GARCH(1,2)	snorm	0.0221	EGARCH(2,2)	snorm	537.4	GARCH(1,3)	norm	194.2066
ETH	2019	EGARCH(3,1)	snorm	1.9E-05	CGARCH(3,3)	ged	0.0026	CGARCH(1,3)	ged	931.9	EGARCH(1,1)	sstd	−8.3864
	2020	EGARCH(2,3)	ged	2.8E-04	EGARCH(2,3)	snorm	0.0035	EGARCH(2,3)	sged	1228.7	EGARCH(3,3)	sstd	−4.7916
	2021	GJRGARCH(2,1)	norm	5.0E-05	GJRGARCH(2,1)	norm	0.0031	GARCH(1,3)	ged	1151.1	EGARCH(3,3)	sstd	−8.2948
LTC	2019	CGARCH(2,2)	snorm	3.8E-05	CGARCH(1,2)	ged	0.0025	-	-	-	EGARCH(3,1)	std	−7.8135
	2020	CGARCH(3,3)	ged	1.5E-04	CGARCH(1,3)	ged	0.0034	CGARCH(1,1)	std	1020.8	GARCH(3,3)	sstd	−4.0872
	2021	EGARCH(2,3)	ged	1.4E-04	CGARCH(2,2)	snorm	0.0042	EGARCH(2,3)	ged	713.6	CGARCH(3,3)	std	−4.6163
XLM	2019	CGARCH(2,1)	snorm	2.4E-05	CGARCH(1,1)	sstd	0.0021	CGARCH(2,2)	ged	1118.2	EGARCH(3,3)	std	−9.9718
	2020	EGARCH(2,3)	norm	1.6E-04	CGARCH(3,1)	sstd	0.0041	CGARCH(2,1)	ged	4127.8	EGARCH(1,1)	std	−1.2865
	2021	EGARCH(2,2)	snorm	3.4E-04	EGARCH(2,2)	sged	0.0054	GJRGARCH(2,2)	norm	581.7	CGARCH(2,1)	sstd	−5.1513
XRP	2019	CGARCH(3,1)	sstd	1.3E-05	CGARCH(1,2)	ged	0.0016	CGARCH(3,1)	sged	812.8	CGARCH(1,1)	norm	−10.0536
	2020	GJRGARCH(2,1)	snorm	4.0E-04	CGARCH(2,1)	sstd	0.0044	CGARCH(2,1)	snorm	23962.6	EGARCH(1,1)	std	−8.2449
	2021	EGARCH(1,1)	sged	2.9E-04	EGARCH(1,1)	ged	0.0066	CGARCH(2,3)	norm	186.9	EGARCH(3,3)	sstd	−4.8562

Note(s): The selected scedastic function, the structure, the error distribution, and the corresponding values of the forecast error metrics (MSE, MAE, MAPE and QLIKE) are given for each digital coin and each out-of-sample period
Source(s): Authors' own work

Particularly for Bitcoin (BTC), the years 2019 and 2020 showed that skew student and skew normal distributions are associated with more accurate forecasts (Table 4). For 2021, EGARCH and GJR-GARCH models, with normal and skewed normal distributions, presented the best volatility forecast results. For ETH, results were similar, as asymmetric GARCH models are better suited, as indicated in Table 4. In the case of XLM and XRP, both digital coins had similar results. CGARCH was the best predictive model in 2019, but for 2020, a clear better approach was not identified. For 2021, EGARCH performed accurately.

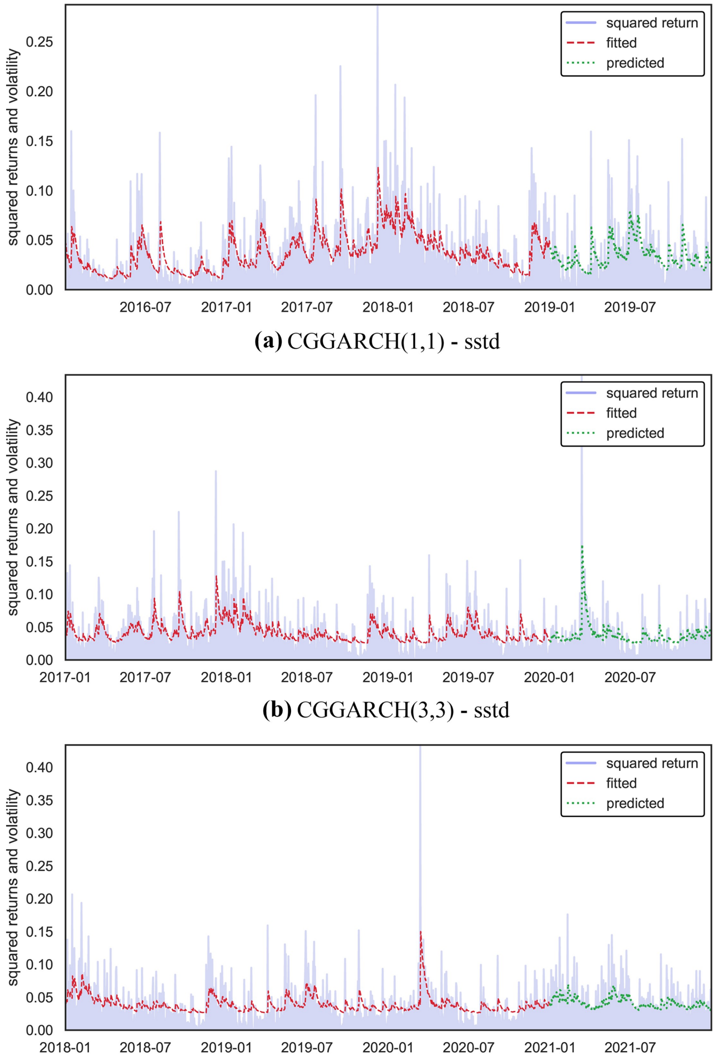
Finally, to illustrate the capability of the methods to model cryptocurrency return volatility, Figure 2 shows the modeling results for Bitcoin, as an example, considering the selected best structure and error distribution in terms of BIC and loss functions, respectively. For each in-sample and out-of-sample set, the temporal evolution of the squared returns (a measure of actual volatility), the estimated volatility (in-sample) and the one-step-ahead volatility forecasts (out-of-sample estimated) are provided. It is possible to verify that the estimated (red line) and predicted (green line) volatility can capture return conditional variance, as they move under the squared returns variation. In addition, during periods of squared returns peaks, the estimates/forecasts can accurately reflect the rise in volatility. A high volatility means that higher values of negative returns, or losses, can be observed, which is of fundamental interest to investors (see Figure 2).

5. Conclusion

This paper performed a robust analysis on the modeling and forecasting of the volatility of price returns for cryptocurrencies using GARCH-family models. Considering data for the period from January 2016 to December 2021, the selected digital coins were Bitcoin, Dogecoin, Ethereum, Litecoin, Stellar and Ripple. Different GARCH models were estimated for each crypto in terms of: the scedastic function (GARCH, GJR-GARCH, EGARCH and CGARCH); error distributions (normal, student's t, generalized error distribution and their corresponding skewed versions); and the model's structure parametrizations (number of parameters). In-sample analysis selected the best models in terms of BIC criteria, whereas one-step-ahead volatility forecasts were evaluated according to different loss functions under different market dynamics.

Generally, for the modeling and forecasting of crypto volatility, the main findings can be summarized as follows. First, parsimonious structures provided better in-sample and out-of-sample accuracy, with a better generalization capability. Second, in terms of error distribution, most of the best results are associated with normal and student's t distributions. Third, concerning the structure of the model, scedastic functions that incorporate volatility asymmetry behavior (EGARCH, especially) provided better volatility estimates and forecasts for cryptocurrencies. Fourth, the best scedastic function is also dependent on the generating process dynamic rather than the period considered, i.e. this selection depends on the digital coin rather than the period under evaluation. Finally, periods associated with systemic events, such as the COVID-19 pandemic, and characterized by a more volatile dynamic, as 2021, negatively impacted the accuracy of the models. Hence, our main findings address the literature on volatility modeling, indicating that parsimonious GARCH-family models are adequate not only for stocks but also for assets with high volatile returns, such as cryptocurrencies.

As a practical implication, the findings suggested that when modeling and forecasting cryptocurrency return volatility, parsimonious asymmetric GARCH models with normal/student distributions provide adequate results in risk management. In addition, investors should monitor the current state of the market before performing the forecasts, as the models could present degraded accuracy in highly volatile periods. A policy implication is the



(a) CGGARCH(1,1) - sstd

(b) CGGARCH(3,3) - sstd

(c) GJRGARCH(1,2) - snorm

Figure 2. Temporal evolution of Bitcoin squared returns (as a proxy of actual volatility), and the corresponding estimated and one-step-ahead predicted volatility using the best fitted GARCH-family model for the in-sample/out-of-sample sets considered in this work
Source(s): Authors' own work

suggestion of parsimonious GARCH techniques for risk management in the cryptocurrency market. This is crucial for providing market practitioners tools for a better measurement of the risk-return trade-off, which is fundamental in investment analysis. Future works include the analysis of GARCH models with the presence of regime switching (Markov switching

GARCH), the construction of GARCH methods integrated with machine learning (ensemble) techniques, the comparison against other volatility approaches such as realized volatility models, and the economic evaluation of the predicted variance in the pricing of cryptocurrency options or in risk measures construction, given that in both cases volatility estimate is a key variable.

Note

[1.] The average price is calculated using volume weighting of quotations provided by over a thousand partner spot exchanges through CoinGecko. Details of the methodology can be found at: www.coingecko.com/en/methodology, Access on 25, October 2024.

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Appendix

Table A1. Descriptive statistics of the predicted errors obtained from the best models selected according to the mean squared error (MSE)

Crypto	Period	Model	Dist.	Mean	SD	Min.	Q 0.25	Q 0.50	Q 0.75	Max.
BTC	2019	CGARCH(1,1)	sstd	−0.011	0.026	−0.062	−0.026	−0.018	−0.003	0.140
	2020	CGARCH(3,3)	sstd	−0.014	0.032	−0.150	−0.029	−0.016	−0.004	0.348
	2021	GJRGARCH(1,2)	snorm	−0.010	0.028	−0.063	−0.031	−0.016	−0.003	0.140
DOGE	2019	CGARCH(1,3)	norm	−0.015	0.024	−0.089	−0.027	−0.018	−0.005	0.105
	2020	CGARCH(2,1)	norm	−0.019	0.042	−0.205	−0.036	−0.023	−0.010	0.360
	2021	EGARCH(1,2)	snorm	−0.023	0.115	−0.492	−0.051	−0.031	−0.004	1.405
ETH	2019	CGARCH(3,3)	ged	−0.021	0.031	−0.103	−0.038	−0.026	−0.005	0.494
	2020	EGARCH(2,3)	snorm	−0.018	0.041	−0.130	−0.038	−0.026	−0.002	0.492
	2021	GJRGARCH(2,1)	norm	−0.011	0.036	−0.084	−0.036	−0.019	0.002	0.241
LTC	2019	CGARCH(1,2)	ged	−0.009	0.030	−0.067	−0.025	−0.017	−0.001	0.225
	2020	CGARCH(1,3)	ged	−0.014	0.040	−0.105	−0.036	−0.022	−0.003	0.408
	2021	EGARCH(2,2)	snorm	−0.015	0.045	−0.087	−0.042	−0.025	−0.003	0.354
XRP	2019	CGARCH(1,1)	sstd	−0.012	0.027	−0.089	−0.027	−0.018	−0.004	0.184
	2020	CGARCH(3,1)	sstd	−0.018	0.048	−0.124	−0.034	−0.024	−0.011	0.375
	2021	EGARCH(2,2)	snorm	−0.017	0.052	−0.141	−0.041	−0.025	−0.001	0.367
XLM	2019	CGARCH(1,2)	ged	−0.013	0.031	−0.083	−0.030	−0.018	−0.004	0.226
	2020	CGARCH(2,1)	sstd	−0.017	0.045	−0.193	−0.038	−0.024	−0.005	0.397
	2021	EGARCH(1,1)	ged	−0.015	0.052	−0.141	−0.041	−0.025	−0.001	0.438

Note(s): *Q* stands for the corresponding quantiles
Source(s): Authors' own work

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